ECONOMIC DYNAMICS: APPLICATIONS TO PUBLIC ECONOMICS, HEALTH ECONOMICS AND LABOR ECONOMICS

Na HAO

A thesis submitted to the Faculty of Graduate Studies
in partial fulfillment of the requirements
for the degree of
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ABSTRACT

The research gap in the literature concerning the application of dynamic economic principles in applied microeconomic framework motivates the present study and involves three related essays. The first essay, entitled *Optimal Dynamic Commodity Taxation*, characterizes the optimal commodity tax in an economy involving the evolution of consumption from an old to a new commodity, represented by a replicator dynamic equation. The paper is motivated by the Canadian Copyright Board introduction of levies on blank audio cassette tapes (old good) and compact discs (new good) in 1999. The study finds that the optimal dynamic tax rate minimizes the discouragement of consumption, and minimizes the impact of the tax on the growth of the consumption of the new good. Also, a dynamic commodity tax is predicted to be selected over a static commodity tax, when the initial proportion of consumers purchasing the new, preferred good in the population is sufficiently small. The study contributes to the literature on optimal commodity taxation, through the application of the concepts of evolutionary game theory to the commodity tax literature.

The second essay, entitled *Government Funding Policy towards Communicable Diseases*, investigates the government choice to offer a production subsidy to a foreign drug monopoly firm producing in a local market aimed at reducing production costs of the drug and thus lowering the local prices for treatment given the prevalence path of the disease in a dynamic economic framework. The study finds that, the foreign monopolist will accept the government funding if its productivity type is sufficiently high. Second, the optimal level of government funding increases with the expected ex-ante cost of production, and decreases with the expected type parameter of the firm. Third, the government would be more likely involved

when the ex-ante cost of production and/or the type parameter of the firm are anticipated to be high, and the number of sick in the population reaches a sufficiently large value.

The third essay, entitled *Search Intensity, Job Offer Arrival Rate and Labor Market Transitions*, presents an empirical structural job search model in which the search effort is endogenized. We use the Canadian data of Labor Market Activity Survey (1988-1990), which contains three indicators of search intensity to investigate the influence of search intensity on the job offer arrival rate and thus on the labor market transitions for those who are unemployed. The estimation results show that all three search indicators have significant impacts on the probability of receiving job offers. Additionally, the incentive effect of insurance compensation system is also evaluated. The simulation results reveal that monitoring and benefit sanctions for insufficient search, may serve as a more effective way to restore incentives for the unemployed workers to get back to work soon.

Dedicated to my parents

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1. OVERVIEW

1.1 INTRODUCTION

This research investigates economic dynamics and its implications for public finance, health economics and labor economics. Economic dynamics has received much more attention in macroeconomics through the resurgence of growth model including endogenous growth and other applications. In microeconomics, the implication of dynamic growth path has been less examined, especially relating to the fields of public economics, health economics and labor. Previous works suggest that dynamic economic equilibrium results and policy implications may differ from those in the static framework. Consequently, there remains a research gap in the literature concerning the application of dynamic economic principles in applied microeconomic framework, including public, health and labor economics. This research gap motivates the present study and involves three related papers. These papers are entitled as follows: optimal dynamic commodity taxation; government funding policy towards communicable diseases; and search intensity, job offer arrival rate and labor market transitions. An outline of each paper is provided below.

1.2 OPTIMAL DYNAMIC COMMODITY TAXATION

This paper characterizes the optimal commodity tax in an economy, involving the evolution of consumption from an old to a new commodity, represented by a replicator dynamic equation. The paper is motivated by the efforts of the Canadian Copyright Board to place levies on blank audio cassette tapes (old good) and compact discs (new substitute product), to recover revenues lost by music copyright owners because of copyright infringement in 1999.

By 2002, the levy had raised 27.8 million in revenues, with the levy rates adjusted each year between 1999 and 2003. The specific questions addressed by this study are as follows: First, what is the dynamic behavior of the optimal commodity tax, both on the new substitute and on old goods, in an economy with consumption replicator dynamics? Second, what impact will consumption dynamics between two goods have on the standard Ramsey Rule? Third, under what conditions will the government choose to implement a dynamic taxation rule, as opposed to a static tax policy? These questions will be addressed through the development of an economic model, involving the government choosing the level of commodity taxation in order to maximize the indirect utility function, subject to a government revenue constraint and a replicator dynamic process. The replicator dynamic process, as in Samuelson (1998) and Hauert et al (2002), represents the evolution in the proportion of the population choosing a strategy to use the new substitute good, as opposed to the old good in a three good economy. The main findings of the study are as follows: First, the optimal commodity tax in the presence of consumption dynamics involves the minimization of the distortion in consumption (as in the standard Ramsey Rule) and the minimization of the impact of the tax on the growth in the proportion of the population consuming the new substitute good. Second, the government chooses to implement a dynamic as opposed to a static taxation scheme, when the initial number of consumers adopting the new technology commodity is sufficiently small. An empirical model is also developed to investigate the welfare and commodity tax implications of the use of a static as opposed to a dynamic framework using examples of new product introduction, along with dynamic commodity taxation. The two examples utilized include: first, the new technology TVs (LCD TVs) and old technology TVs (Tube TVs); and second, blank CD-Rs and blank audio cassettes used for recording and listening to music. The empirical results are consistent with predictions implied by the theoretical model. This paper contributes to the literature on optimal commodity taxation, through the application of evolutionary game theory concepts to the commodity tax literature.

1.3 GOVERNMENT FUNDING POLICY TOWARDS COMMUNICABLE DISEASES

The paper is motivated by the prevalence of communicable diseases such as malaria, tuberculosis and HIV/AIDS in developing countries, which may involve monopoly powers in the provision of pharmaceutical drug, and coupled with government concern about the welfare of individuals in the population. The study investigates the government choice of offering a production subsidy to a foreign drug monopoly firm producing in the local market. The subsidy is aimed at reducing production costs of the drug, and thus lowering the local prices for treatment, given the prevalence path of the disease in a dynamic economic framework. The specific research questions addressed by this study are as follows: First, under what conditions will the drug monopolist choose to accept the offer of the local government, and how does the monopoly price for treatment behave when the number of sick in the population grows? Second, what parameters will influence the optimal level of government funding and what are the directions of change? Third, under what conditions would the local government offer the subsidy for reducing production cost for treatment to the foreign drug monopoly in the market? These questions will be addressed through the development of an economic model, involving both the government choice and the monopolist's choice, in a dynamic environment given the prevalence path of the disease. The economy is represented by a sequential game involving: In stage 1, the government chooses whether or not to offer a production subsidy to the monopolist. If the government decides to do so, the optimal value of the government funding is determined at stage 2. In stage 3, the monopolist chooses whether or not to accept the offer provided by the government, by comparing its profit with and without government subsidy. If the firm accepts the offer, the dynamic prices for treatment are determined by the monopolist, given the ex-post cost of production starting at stage 4. Otherwise, if the firm decides to reject the government funding, the market prices for treatment are determined by the monopolist given the ex-ante cost of production from stage 4. Whereas, if the government decides not to provide the fund in the first stage, the monopolist in the market producing at the ex-ante cost, determines the dynamic prices for treatment beginning from stage 2. Given the market prices for treatment, consumers choose whether or not to purchase the drug, affecting both the individual's chances of recovering from the disease, as well as its communicability to other individuals in the economy. The paper therefore extends Mechoulan (2007) framework, which considers treatment externalities of communicable diseases, under different market structures, and characterizes the optimal price and prevalence paths in an dynamic framework. This is addressed via an agent-based model through the introduction of the government, and the monopoly's choice whether or not to accept the government production subsidy aimed at reducing the cost of production for the pharmaceutical drug. The key findings of the paper are as follows: First, the foreign drug monopolist takes the production subsidy of the local government, if its productivity type parameter is sufficiently high. Additionally, the monopoly price for treatment declines with the prevalence of the disease. Second, when the optimal value of government funding is greater than zero, the optimal level of the subsidy increases with the expected ex-ante cost of production, and decreases with the expected type parameter of the firm. Third, the local government would be more likely to be involved by providing production subsidy to the foreign drug monopolist when the ex-ante cost of production and/or the type parameter of the firm, are anticipated to be high and the number of sick in the population reaches a sufficiently large value. The paper also extends the paper by Mechoulan (2007) in demonstrating theoretical results, through the application of a computational dynamic optimization model. The paper contributes to the literature regarding the economics of pharmaceutical drug production for communicable diseases in an economy with market power, externalities and heterogenous agents, by introducing a government with the capacity to tax and subsidize the local production of drugs aimed at reducing the production cost of drugs, thus lowering the prevalence of the disease. Additionally, the framework is extended to characterize the equilibrium outcomes when the government subsides a potential new entrant firm, as opposed to the incumbent.

1.4 SEARCH INTENSITY, JOB OFFER ARRIVAL RATE AND LABOR MARKET TRANSITIONS

This paper investigates the impact of search intensity of individuals who are unemployed on the transition rate into employment by influencing the job offer arrival rate using the longitudinal data file (1988-1990) of Labor Market Activity Survey of Canada. An empirical structural model of job search with endogenous search effort is presented and estimated in a stationary framework based on the model by Mortensen (1986). This study extends Mortensen's (1986) job search model by the following ways: First, we add a constant term to capture the transition into work of non-searchers. Second, we define search effort as a vector of three different search indicators: one for "search or not" and two for "search intensity" (as opposed to a one-dimensional search intensity used in the original model). Third, we specify the job offer arrival rate as a function of the composite sum of various indicators of search intensity. In contrast, in Mortensen's (1986) model, the arrival rate of job offers depends on the single-dimensional overall search effort only. The Canadian data of Labor Market Activity Survey (1988-1990) contains information on the labor market participation and job characteristics of all responding individuals over three year period. Therefore, this data set provides us with the opportunity to track the working history, job search information and the transition into a new job between calendar years, for those who reported to experience unemployment over the three year survey period. The estimation results show that all three indicators of search do influence the job offer arrival rate significantly and the unemployed with a higher level of search effort and a relatively lower value in reservation wage transit into work sooner compared with the others. To evaluate the incentive effects of the insurance compensation on the behavior of job search and thus on the labor market transition rate, we simulate two alternative policy reforms on the unemployment insurance system (i.e., a one time permanent benefits cut, and monitoring and benefits sanction for insufficient search), using the parameters of the structural model. The simulation result reveals that compared with the permanent benefits cut, the monitoring and benefit sanction for insufficient search seems to be a better way to provide incentives for the unemployed workers to search more, demand less and get back to work sooner. The study contributes to the literature on job search models with endogenous search effort, by investigating the impacts of search intensity on job finding success in an empirical structural job search model which is stationary using the longitudinal Canadian data containing three indicators of search intensity. The optimal strategy of job seekers in a non-stationary framework where exogenous variables are changing over time is also examined. Additionally, the equilibrium results are characterized for the optimal search intensity and optimal reservation wage in a non-stationary environment.

1.5 CONCLUSION

The study suggests that economic dynamics is an important consideration in the public sector choice of appropriate commodity tax policy, subsidizing the local production of drugs in the presence of market power and externality due to communicable diseases, and in determining the optimal job search strategy in labor markets. The incorporation of optimal dynamic agent behavior in microeconomic models has non-trivial welfare implications, that suggest the over use of static microeconomic model in public finance may mask opportunities for welfare improvement through the implementation of dynamic as opposed to static policy choices. The study therefore extends the traditional Ramsey static model for optimal commodity taxation to a dynamic framework with an evolutionary game. Additionally, the Mechoulan (2007) model is extended to incorporate the government's role in curtailing the dispersion speed of a communicable disease, when a monopolist supplies a needed pharmaceutical drug. The extension demonstrates that the government's role may be increasingly appropriate as the prevalence of a communicable disease rises. Furthermore, in a job search environment, the insurance compensation has incentive effects on the optimal strategy of job seekers in a dynamic as opposed to static framework. These results provide urgency for further work in the application of dynamic economic choices in public economics, health economics and labor economics.

2. OPTIMAL DYNAMIC COMMODITY TAXATION

2.1 INTRODUCTION

This paper characterizes the optimal commodity taxation in an economy involving an evolutionary equilibrium, reflecting a consumption replicator dynamics, and provides a numeric dynamic optimization analysis of the related welfare implications. Consumption replicator dynamics can be considered as the adoption of a new substitute good that results in changes in the proportion of the population consuming the new substitute good as opposed to the old good.[1] There are numerous examples of the development of new substitute products, such as: color televisions, microwaves, computers, and compact discs. These products coexist with older goods. The characterization of the optimal commodity taxation for static economies (i.e., Ramsey Rule) has been completed by Ramsey (1927), Dixit (1970), Atkinson and Stiglitz (1972), Diamond (1975), Mirrlees (1976), and, more recently, Coady and Dreeze (2002). For dynamic economies, the Ramsey Rule has been characterized for neoclassical growth and endogenous growth models (Jones, *et al*, 1993, and Coleman II, 2000). However, the dynamics involved in an evolutionary equilibrium reflective of the introduction of a new good in a market has received little attention in the tax literature.

The study is motivated by the efforts of the Canadian government and others to place levies on blank audio cassette tapes (old good) and compact discs (new substitute product) to recover revenues lost to music copyright owners through copyright infringement.[2] In 1999, the Copyright Board of Canada implemented levies of 23.3 cents on blank audio cassette tapes, 5.2 cents on CD-R and CD-RW, and 60.8 cents on CD-R and CD-RW Audio. These

[1] Samuelson (1998) summaries the replicator equation for very short time periods, t, as follows: $dx_i/dt = x_i\{\pi(i,x) - \overline{\pi}(x)\}$, where: $\pi(i,x)$ is the payoff to player i for choosing the mixed strategy x and $\overline{\pi}(x) = \sum_{i \in S} x_i \pi(i,x)$. S is the set of pure strategies. x can be interpreted as the proportion of consumers choosing a pure strategy (e.g., consumption of the new substitute good).

[2] Other examples include: new LCD and old tube television, combustion engines and electric or hybrid engines, and land line and cellular telephones.

rates were adjusted in 2002 and again in 2003. In 2003, these rates were set at 29 cents for tapes, 21 cents for CD-R/RW, and 77 cents for CD-R/RW Audio (Copyright Board of Canada, 1999 and 2002).[3] The revenues raised through the levies reached $24.1 million in 2001 and $27.8 million in 2002. Baker (1992) also provides an overview and analysis of the British government's 1986 proposal to place a levy on blank audio tapes. The characterization of an optimal commodity taxation scheme for an economy with consumption replicator dynamics has yet to be addressed in the tax literature. This study addresses this gap.

The specific questions addressed by this study are as follows: First, what is the dynamic behavior of the optimal commodity tax, on the new substitute and old goods, in an economy with consumption replicator dynamics? Second, what impact will consumption dynamics between two goods have on the standard Ramsey Rule? Third, under what conditions will the government choose to implement a dynamic taxation rule, as opposed to a static tax policy? These questions will be addressed through the development of an economic model, which involves the government choosing the level of commodity taxation in order to maximize the indirect utility function, subject to a government revenue constraint and a replicator dynamic process. The replicator dynamic process reflects the dynamics between the proportion of the population choosing to use the new substitute good as opposed to the old good. The three goods in the economy are identified as follows: a new substitute good (e.g., LCD TV and compact disc), an old good (e.g., tube TV and blank audio cassettes) and a composite good. The economy is represented by a sequential game involving: The government chooses between an optimal static and dynamic taxation scheme at stage 1. At stage 2, the government chooses the commodity tax level to maximize social welfare, consisting of the aggregate of the individual indirect utility function of the type 1 and type

[3]On December 23, 2003, the Copyright Board of Canada froze the levy rates for 2004 at the 2003 levels.

2 consumers, given a revenue constraint. For the dynamic subgame, a replicator dynamics constraint is added to represent the evolutionary game process from the old good to the new good. Type 1 consumers utilize the new good and have a non-zero probability of interacting with a consumer of the same type yielding a positive externality if so and no externality otherwise. Type 2 consumers utilize the old good, and gain a positive externality if interacting with their own type and no externality otherwise. Type 1 and Type 2 consumers are involved in an evolutionary game at stage 3, resulting in a replicator dynamics process. Type 2 consumers may decide to deviate to adopt the new technology good if they could achieve an above average expected utility by choosing new product. At stage 4, consumers choose the consumption level of each good observing the prices and the commodity tax rates. The consumption dynamics reflect the dynamic adjustment away from one good (e.g., old good) towards another good (e.g., new good) based on consumer choice. This process is affected by prices, so a commodity tax affects the proportion of the population consuming each of these goods. Additionally, the government can choose to implement a dynamic or static tax policy. For simplicity, the switching cost over time incurred by the government under a dynamic tax policy is assumed to be zero.

The main findings of the study are as follows: First, the optimal commodity tax in the presence of consumption dynamics involves the minimization of the distortion in consumption (as in the standard Ramsey Rule), and the minimization of the impact of the tax on the growth in the proportion of the population consuming the new substitute good. The dynamic commodity tax is shown to be inversely related to the impact it has on the growth in the consumption of the new substitute good. It therefore follows that the tax rate on the substitute good rises over time and decreases for the old commodity as the product cycle of the new substitute good moves from a newly introduced to a mature product. Once the new product is fully adopted, the economy achieves an evolutionary stable equilibrium.

The old product is out of the market and the tax rate at equilibrium for new commodity is determined by the standard Ramsey Rule. Second, the government chooses to implement a dynamic as opposed to static taxation scheme, when the initial number of consumers adopting new technology good is sufficiently small. Therefore, the government can encourage the consumers to adopt the newly developed commodities and improve aggregate social welfare with the implementation of the dynamic taxation policy.

Literature Review. The study relates to three areas in the literature: commodity taxation in a static economic environment, commodity taxation in a dynamic economic environment, and evolutionary games. First, the examination of optimal commodity taxation has primarily emerged out of the work initially conducted by Ramsey (1927). In a static economic framework, the Ramsey Rule characterizes the optimal commodity tax for a government choosing the level of tax to maximize the sum of the agents' indirect utility given a revenue constraint. Dixit (1970) provides an early consolidation of the optimal commodity tax problem. Atkinson and Stiglitz (1972) examine optimal commodity taxation in the presence of savings, risk-taking and leisure. Diamond (1975) examines the problem in a multi-person economy. Mirrlees (1976) develops an index of discouragement for commodities that provides a clear linkage between the Ramsey Rule and setting commodity taxation to minimize deadweight loss or distortion in consumption. Recently, Coady and Dreeze (2002) examine commodity taxation in a general equilibrium framework, contributing to the literature on the generalized Ramsey Rule that was developed earlier by Guesnerie (1979) and Drèze and Stern (1987).

Second, in dynamic frameworks, authors have used standard neoclassical and endogenous growth models to characterize optimal taxation rules. Jones, Manuelli and Rossi (1993) provide an overview of this literature. Krusell *et al* (1996) suggest that tax rates in a dynamic economy are not determined once, but rather are continually changing over time.

Coleman II (2000) investigates the welfare gains associated with switching from a static to a dynamic tax policy in a deterministic discrete, infinite time horizon economy. Substantial welfare gains are predicted from switching to a dynamic Ramsey tax policy. These results are consistent with early work done by Jones et al (1993).

Third, the evolutionary game literature provides a framework for considering the dynamic transition from an old commodity to a new substitute good (e.g., consumption dynamics). Specifically, agents may learn and adopt a new strategy when the new strategy yields a payoff that is above the average payoff (Gardner, 1995; Samuelson, 1998; Cressman *et al*, 1998; and Witt, 2001). The proportion of the population using the new strategy increases over time under these circumstances and is represented by replicator dynamics or a replicator equation. Hauert *et al* (2002) also provides an extensive overview of the replicator equation. The affect of taxation on substitution between goods has often been considered through the marginal rate of substitution or the marginal rate to technical substitution (Scalera, 1995, and Zou and Gong, 2002). Slive and Bernhardt (1998) use the impact of the price level and the impact of a network externality, on the proportion of the population conducting piracy, in their investigation of the enforcement of copyright laws in the computer software industry in a static economic framework. Network externality being the positive externalities associated with many individuals using the same software.[4] The characterization of an optimal tax rule in an economy with consumption dynamics remains a gap in the literature.

The paper proceeds as follows: The theoretical model is outlined in section 2 of the paper. In section 3, the optimal commodity taxation levels are determined for the static subgame first and then for the dynamic subgame. In addition, the characterization of the government's choice to implement a static or dynamic taxation rule is also investigated in section 3. In section 4, empirical application of the theoretical model in section 4 is utilized

[4]Metcalfe (2001) work on consumption, preferences, and the evolutionary game suggests that network interaction and externalitis are important in the dynamics of the evolutionary process.

to investigate the welfare and commodity tax implications of the use of a static as opposed to a dynamic framework. The conclusion of the study is outlined in section 5,. The study contributes to the literature on optimal commodity taxation in presence of consumption dynamics.

2.2 THE MODEL

Consider an economy consisting of two types of individuals, a government and consumption goods $x_1(t)$, $x_2(t)$ and $x_3(t)$ at time t. Consumers choose the level of consumption goods to maximize their utilities given prices, income and commodity taxes. Consumers may also choose to switch between the consumption of a new and a old commodity. The government chooses the commodity tax rates to maximize social welfare given a revenue constraint. The government may choose either a static or a dynamic commodity tax scheme. In what follows, the game is formally described starting with consumers and followed by the government. The choices of each player and sequence of events are described prior to the game being solved. Perfect information is assumed throughout the game.

Consumer 1 has preferences over goods $x_1(t)$ and $x_3(t)$ while consumer 2 has preferences over $x_2(t)$ and $x_3(t)$. Good $x_1(t)$ can be considered to be a new substitute such as a LCD television or compact disc; $x_2(t)$ can be considered to be an old product, such as a tube television or a blank audio cassette tape; and $x_3(t)$ is a composite of the other goods in the economy. The consumer faces prices p_1, p_2, and p_3, respectively, and has income denoted by $w > 0$. It is assumed that prices and income are constant over time. Commodity taxes $\tau_1(t)$ and $\tau_2(t)$ are placed on goods $x_1(t)$ and $x_2(t)$ by the government to raise revenues $R > 0$ at t. The consumers' preferences are represented by the indirect utility function given by $V_1(p_1 + \tau_1(t), p_3, w)$ and $V_2(p_2 + \tau_2(t), p_3, w)$ for consumers of type 1 and of type 2, respectively. The number of type 1 consumers in the population at time t is given by $N_1(t)$, the number of type 2 consumers is given by $N_2(t)$ and the total population of consumer in the economy is denoted by $N > 1$.

The economy is represented by a symmetrical evolutionary game with two players and two strategies each (see Figure 2.1). Consumers adopting the new substitute good are considered to be type 1 consumers and otherwise they are labeled type 2. Additionally, consumers are assumed to experience a positive externality when encountering another individual of the same type. For instance, the consumers may be able to share music and video files if media recording and playing devices are utilized. This externality is expressed by $e_1 > 0$ for type 1 consumers and $e_2 > 0$ for type 2 consumers such that consumer i's payoff is $V_i(p_i + \tau_i(t), p_3, w) + e_i$ if they encounter a type i player and $V_i(p_i + \tau_i(t), p_3, w)$ otherwise, $i = 1, 2$.

The assumption about preferences and the externality is made as follows:

Assumption **A1**: Preferences and the externality are assumed to satisfy: $V_1(p_1+\tau_1(t), p_3, w) < V_2(p_2 + \tau_2(t), p_3, w) + e_2 < V_1(p_1 + \tau_1(t), p_3, w) + e_1$ with $e_1 > e_2$

A1 implies that there are two pure strategy equilibria for this 2 by 2 symmetrical game and the payoff at the equilibrium (Good 1, Good 1) dominates that at the equilibrium (Good 2, Good 2). The externality incurred by using the new substitute good is assumed to be greater than that incurred by choosing the old product. The new substitute good $x_1(t)$, provides the consumers with a higher utility when all consumers adopt the new technology good, than that provided by the old product $x_2(t)$ when all consumers choose the old commodity. $x_1(t)$ and $x_2(t)$ can be interpreted as inputs used in household production. The technology using $x_1(t)$ and $x_2(t)$ is assumed to differ and each household is assumed to possess only a single technology.

Let $a(t) \in [0, 1]$ denote the percentage of the population choosing a strategy of purchasing good x_1 at time $t \geq 0$ and $1 - a(t)$ denote the percentage of the population choosing a strategy of purchasing good x_2 at time t. The expected utility for consumers purchasing the new substitute good x_1 at time $t+1$ is therefore $U_1 = a(t)\left[V_1(p_1+\tau_1(t),p_3,w)+e_1\right] + [(1-a(t)]V_1(p_1+\tau_1(t),p_3,w) = V_1(p_1+\tau_1(t),p_3,w)+a(t)e_1$. The expected utility for consumers purchasing the old product x_2 at time $t+1$ is $U_2 = a(t)\left[V_2(p_2+\tau_2(t),p_3,w)\right]+[(1-a(t)]\left[V_2(p_2+\tau_2(t),p_3,w)+e_2\right] = V_2(p_2+\tau_2(t),p_3,w)+(1-a(t))\,e_2$. It is useful to define the average utility in the population that is calculated as $\bar{U} = a(t)U_1+(1-a(t))U_2$. Using Roy Gardner (1995), the equation for replicator dynamics is set up as $\dfrac{\partial a(t)}{\partial t} = a(t)\left[U_1-\bar{U}\right] = a(t)(1-a(t))\left[U_1-U_2\right]$. Since $U_1 = V_1(p_1+\tau_1(t),p_3,w)+a(t)e_1$ and $U_2 = V_2(p_2+\tau_2(t),p_3,w)+(1-a(t))\,e_2$, we denote the replicator equation which captures the dynamics in the population adopting good $x_1(t)$ over time as $a'(t) = g(a(t),p_1+\tau_1(t),p_2+\tau_2(t),p_3,w)$ in the sequel. The evolution in consumption associated with the movement in the proportion of the population learning about and adopting $x_1(t)$ as opposed to consuming $x_2(t)$ over time can be represented by the replicator dynamics in an evolutionary game.[5] According to the replicator dynamics, type 2 consumers would consider switching to good x_1 instead of choosing good x_2 at time $t+1$ if the new technology could provide them with an above-average utility meaning $U_1 > \bar{U}$. As a result, the proportion of the population adopting new product would increase at $t+1$ implying $a'(t) > 0$. However, if the expected utility by choosing good x_1 was lower than that by choosing good x_2 meaning $U_1 < \bar{U}$ at time $t+1$, type 2 consumers would not deviate and the type 1 consumers would like to choose the old product as opposed to adopt the new substitute good implying $a'(t) < 0$. The phase diagram which captures the evolution in $a(t)$ is plotted in Figure 2.2.

[5]The replicator dynamics says that if a player type earns an above average payoff, then its percentage in the population increases; while if a player type earns a below average payoff, then its percentage in the population decreases. The below-average player types will learn to copy the strategy of above-average player types over time. Since they are slow learners, not all player types earning the below-average payoff will switch all at once, but eventually all player types still present in the population will earn the average payoff. The learning ceases and the percentage in the population of any player type remains constant (Roy Gardner,1995).

Definition 2.1. Stability Theorem. If $da(t)/dt = F(a^*(t)) = 0$, and $dF(a^*(t))/da(t) < 0$, then $a^*(t)$ is dynamically stable.

Graphically, the stability theorem states that if the replicator equation is downward sloping at a root $a^*(t)$ of the equation, then that root is an evolutionary stable strategy, or ESS for short. Therefore, an equilibrium is an evolutionary stable strategy (ESS), when two things happen: the replicator dynamics points towards this equilibrium and low-probability mistakes do not destroy the stability. The replicator equation we defined above $a'(t) = a(t)(1 - a(t))\,[U_1 - U_2]$ has three roots, which can be derived by setting each of the three factors equal to zero: $a(t) = 0$, $1 - a(t) = 0$ and $U_1 = U_2$. It follows that the corresponding roots are: $a_1^*(t) = 0$, $a_2^*(t) = 1$ and $a_3^*(t) = [V_2(\cdot) + e_2 - V_1(\cdot)]\,/(e_1 + e_2)$ respectively where $V_i(\cdot) = V_i(p_i + \tau_i(t), p_3, w)$ for $i = 1, 2$. The slope of the replicator equation evaluated at each root is derived as follows: $dF(a_1^*(t))/da(t) = V_1(\cdot) - [V_2(\cdot) + e_2]$, $dF(a_2^*(t))/da(t) = -[V_1(\cdot) + e_1 - V_2(\cdot)]$ and $dF(a_3^*(t))/da(t) = a_3^*(t)[1 - a_3^*(t)](e_1 + e_2)$. Apparently, the slopes $dF(a_1^*(t))/da(t)$ and $dF(a_2^*(t))/da(t)$ are both negative by Assumption **A1**. The roots $a_1^*(t) = 0$ and $a_2^*(t) = 1$ are both ESSs. By contrast, the slope $dF(a_3^*(t))/da(t)$ is positive, given Assumption **A1** implies that $a_3^*(t) \in (0, 1)$ and $e_1 > e_2 > 0$, so the root $a_3^*(t) = [V_2(\cdot) + e_2 - V_1(\cdot)]\,/(e_1 + e_2)$ is not evolutionary stable. The replicator dynamics points away from this equilibrium. Figure 2.2 shows that the unstable root at $a_3^*(t) = [V_2(\cdot) + e_2 - V_1(\cdot)]\,/(e_1 + e_2)$ divides the interval $[0, 1]$ into two zones. To the left of $a_3^*(t)$, learning process heads toward to $a_1^*(t) = 0$. To the right of $a_3^*(t)$, learning heads toward to $a_2^*(t) = 1$. Apparently, where learning ends up crucially depends on its starting point. If the initial proportion of type 1 consumers in the population is less than $a_3^*(t) = [V_2(\cdot) + e_2 - V_1(\cdot)]\,/(e_1 + e_2)$, then the population heads toward the ESS where every consumer chooses the old product (i.e., $a_1^*(t) = 0$). Otherwise, the population heads toward the ESS where $a_2^*(t) = 1$. The payoff at the equilibrium where every consumer adopts the new substitute good $x_1(t)$ dominates the payoff at the equilibrium where every consumer uses the old product $x_2(t)$ by Assumption A1, so $a_2^*(t) = 1$ is the only efficient ESS of this game.

The assumption about the initial value of the population consuming good $x_1(t)$ and the direction of learning is made as follows:

Assumption **A2**: It is assumed that the initial value of $a(t)$ satisfies: $a(t_0) \geq a_3^*(t) = \left[V_2(\cdot) + e_2 - V_1(\cdot)\right]/(e_1 + e_2)$ and $a'(t) \geq 0$ for $t \geq t_0$ where: $t_0 = 0$ and $V_i(\cdot) = V_i(p_i + \tau_i(t), p_3, w)$, $i = 1, 2$.

A2 implies that the learning process only takes place when the type 2 consumers copy the type 1 consumers. Upon consumers of type 2 deviating to consume good $x_1(t)$, they stay as type 1 consumers forever thereafter. Clearly, given $a(t_0) \geq a_3^*(t)$, the population heads toward to the efficient ESS, $a_2^*(t) = 1$, in this game.

The government cares about social welfare, and social welfare is assumed to be of the Benthamite social welfare form, represented by: $W(U_1, U_2) = a(t)\left[V_1(p_1 + \tau_1(t), p_3, w) + a(t)e_1\right] + (1 - a(t))\left[V_2(p_2 + \tau_2(t), p_3, w) + (1 - a(t))e_2\right]$. The government focuses on a budget constraint defined by revenue requirements $R(t)$ and tax receipts given by $\sum_{i=1}^{2} N_i(t)\tau_i(t)x_i(q_i(t), p_3, w)$ for period $t \in [t_0, T]$.

The sequence of events for the symmetric evolutionary game is as follows: At stage 1, the government chooses whether or not to implement a static or dynamic optimal commodity tax. At stage 2, the government determines the level of the optimal dynamic commodity taxes by choosing $\tau_1(t)$ and $\tau_2(t)$ given a budget revenue requirement $R(t) > 0$ and a replicator dynamic equation for the dynamic subgame or the optimal commodity taxes on goods x_1 and x_2 using the standard Ramsey Rule for the static subgame. Consumers are involved in an evolutionary game in stage 3. Type 2 consumers choose whether or not to deviate to new technology for the next time period by comparing the expected utility purchasing good

x_1, U_1, and the average utility in the population, $\bar{U}$, given the proportion of the population choosing a strategy of purchasing good x_1 at time $t \geq 0$, $a(t)$. Once consumer 2 has deviated to consume good $x_1(t)$, this consumer is deemed to be a type 1 consumer, and gains the expected indirect utility $U_1(t)$. Otherwise, consumer 2 purchases only good $x_2(t)$, remains a type 2 consumer and gains the expected indirect utility $U_2(t)$. At stage 4, consumers choose the level of the goods to consume given prices, taxes, income, and the technology they possess.

2.3 OPTIMAL COMMODITY TAXATION

In this section, the taxation game is solved through backwards induction starting at stage 4. At stage 4, each consumer chooses the level of the goods to purchase given prices, taxes, and income gaining payoffs $V_1(p_1+\tau_1(t), p_3, w)$ for type 1 consumers and $V_2(p_2+\tau_2(t), p_3, w)$ for type 2 consumers. Consumers are involved in an evolutionary game at stage 3. By Assumption A2, given $a(t_0) \geq a_3^*(t)$, there should exist some tax rates at time t_0 making $U_1 > U_2$ at time $t_0 + 1$. As a result, type 2 consumers would consider deviating to adopt the new technology at $t_0 + 1$ thus making the proportion of population consuming good x_1 increase as $a'(t) = a(t)\left[U_1 - \bar{U}\right] = a(t)(1-a(t))\left[U_1 - U_2\right] > 0$. Note that when the tax rates are set such that $U_1 = U_2$ at time $t = t_0$, the deviation will not stop since type 2 consumers can earn more at $t_0 + 1$ by switching to adopt good $x_1(t)$ at t_0 as shown in Appendix A.1. Given $U_1(t) > U_2(t)$ for $t > t_0$, the learning process will continue up to some time $t = t'$ at which all consumers choose the new technology good, and earn the average payoff $\bar{U} = U_1$ (i.e., $a(t) = a_2^*(t) = 1$ at $t = t'$). The economy therefore achieves an evolutionary stable equilibrium where $a(t) = a_2^*(t) = 1$ at $t = t'$. Correspondingly, the tax rate for the new substitute good remains constant for $t \geq t'$.

Proposition 2.2. *Given **A1** and **A2**, if the tax rates are set such that there exists an equitable allocation, defined by $U_1(t) = U_2(t)$ at some time t, then the game achieves an equilibrium that is an evolutionary unstable strategy.*

The proof of the proposition is given in Appendix A.1. The proposition implies that an equity level of utility can be achieved at time t from the dynamic tax problem with $U_1(t) = U_2(t)$. However, this equilibrium can not be maintained over time as the type 2 consumers could obtain a higher level of expected utility at next period by deviating at time t. Therefore, the proportion of population consuming new substitute good continues to evolve at t (i.e., $a'(t) > 0$). Given Assumptions **A1** and **A2**, the tax rates $\tau_1(t)$ and $\tau_2(t)$ at which $U_1(t) = U_2(t)$ can therefore only be observed when $a(t_0) = a_3^*(t)$ at time $t = t_0$. This initial value of proportion of population consuming good $x_1(t)$ would make the dynamic tax rate equal to the static tax rate for a single instance (i.e., for $t = t_0$). Then, the subsequent dynamic tax rates would point away for all other periods (i.e., for $t > t_0$).

At stage 2, for the static subgame, the optimal commodity tax is defined by the Ramsey commodity tax problem given the initial proportion of type 1 consumers at time t_0. While for the dynamic subgame, the government determines the optimal dynamic commodity tax levels on the two goods, subject to the revenue target and the replicator dynamics equation at stage 2. In what follows, the static subgame is first solved and the dynamic subgame is solved next to determine the efficient evolutionary stable equilibrium. The government's choice between static and dynamic commodity taxation takes place at stage 1 of the game.

2.3.1 STATIC SUBGAME

In the static subgame, given $a(t_0)$, the initial proportion of consumers purchasing new technology good at time t_0, the optimal static commodity taxes are those that give the highest level of social welfare while ensuring that the government reaches its revenue target of $R(t) > 0$ at time t_0. At stage 2, the government's problem in choosing the tax rates can be summarized as follows:

$$(2.1) \qquad \max_{\{\tau_1,\tau_2\}} \{a(t_0)\,[V_1(q_1,p_3,w)+a(t_0)e_1]+(1-a(t_0))\,[V_2(q_2,p_3,w)+(1-a(t_0))e_2]\}$$
$$s.t.\ (1)\ \sum_{i=1}^{2} N_i\tau_i x_i(q_i,p_3,w) \geq R(t)$$

where: $q_i = p_i + \tau_i$ for $i = 1, 2$.

The optimal commodity taxes τ_1^* and τ_2^* are as follows:[6]

$$(2.2) \qquad \tau_1^* = \frac{-\theta_1 x_1(q_1,p_3,w)}{S_{11}}$$

$$(2.3) \qquad \tau_2^* = \frac{-\theta_2 x_2(q_2,p_3,w)}{S_{22}}$$

The results determined for τ_1^* and τ_2^* are effectively the standard Ramsey Rule, implying that the optimal static commodity taxes are set to minimize the distortion in the consumption of each good. Assumption **A2** implies that given $a(t_0) \geq a_3^*(t)$, there should exist some τ_1 and τ_2 such that $U_1(t) > U_2(t)$ at time t_0+1. Therefore, the number of consumers adopting the new technology good grows implying $a'(t) > 0$ at time t_0+1 even though the commodity tax rates remain constant (static) over the time period. According to evolutionary game theory, the learning process continues up to some time $t = t'$ at which all consumers adopt

[6]Let $\alpha_i = \partial V_i(q_i,p_3,w)/\partial w$ and $S_{ij} = \partial x_i(q_i,p_3,w)/\partial p_j$ is the substitution effect for commodity $j \in (1,2)$. In this case, $S_{21} = S_{12} = 0$ since $x_i(q_i,p_3,w)$, for $i = 1,2$, as in the standard Ramsey Rule (Atkinson and Stiglitz, 1972). Furthermore, $\theta_1 = \left[1 - \frac{\alpha_1}{\lambda N} - \tau_1 \frac{\partial x_1(q_1,p_3,w)}{\partial w}\right] > 0$ and $\theta_2 = \left[1 - \frac{\alpha_2}{\lambda N} - \tau_2 \frac{\partial x_2(q_2,p_3,w)}{\partial w}\right] > 0$, where: λ is the Lagrangian multiplier associated with the government's budget constraint.

the new technology goods. Assume the economy achieves the evolutionary stable equilibrium where $a(t) = a_2^*(t) = 1$ under static tax rates at time $t = t_s'$. For $t \geq t_s'$, the static tax rates at equilibrium for $a(t) = 1$, $\tau_1 = \tau_1(t')$ and $\tau_2 = 0$ apply.[7] The subgame equilibrium payoff for the government is thus defined by $W(\tau_1^*(t'), \tau_2^*(t')) = V_1(p_1 + \tau_1^*(t'), p_3, w) + e_1$.

2.3.2 DYNAMIC SUBGAME

In the dynamic subgame, given $a(t_0) \geq a_3^*(t)$ at time $t = t_0$, there should also exist some dynamic tax rates $\tau_1(t)$ and $\tau_2(t)$ such that $U_1(t) > U_2(t)$ for $t > t_0$. To simplify the notation, let $g(a(t), p_1 + \tau_1(t), p_2 + \tau_2(t), p_3, w)$ denote the replicator equation which captures the dynamics in the proportion of the population adopting good $x_1(t)$ over time (i.e., $a'(t) = a(t)\left[U_1 - \bar{U}\right] = a(t)(1-a(t))\left[U_1 - U_2\right]$). Since $U_1 = V_1(p_1+\tau_1(t), p_3, w)+a(t)e_1$ and $U_2 = V_2(p_2+\tau_2(t), p_3, w) + (1-a(t))\,e_2$, the derivatives of $g(\cdot)$ with respect to $\tau_1(t)$ and $\tau_2(t)$ are as follows: $\dfrac{\partial g(\cdot)}{\partial \tau_1(t)} = a(t)(1-a(t))\left[\dfrac{\partial V_1(q_1(t), p_3, w)}{\partial q_1(t)}\right]$ and $\dfrac{\partial g(\cdot)}{\partial \tau_2(t)} = -a(t)(1-a(t))\left[\dfrac{\partial V_2(q_2(t), p_3, w)}{\partial q_2(t)}\right]$. Given $\dfrac{\partial V_i(q_i(t), p_3, w)}{\partial q_i(t)} < 0$ for $i = 1, 2$, it is easily to derive that $\dfrac{\partial g(\cdot)}{\partial \tau_1(t)} < 0$ and $\dfrac{\partial g(\cdot)}{\partial \tau_2(t)} > 0$ for $a(t) \in (0,1)$. The following proposition can be expressed from these results.

Proposition 2.3. *Given* ***A1*** *and* ***A2****, if there exists $\tau_1(t)$ and $\tau_2(t)$ such that an efficient evolutionary stable equilibrium is achieved at time $t = t'$, then $\dfrac{\partial g(\cdot)}{\partial \tau_1(t)} < 0$ and $\dfrac{\partial g(\cdot)}{\partial \tau_2(t)} > 0$ for $t < t'$, while $a'(t) = g(\cdot) = 0$ and $a(t) = 1$ for $t \geq t'$, where: $t' \in (0, T)$.*

The proof of the proposition flows directly from the analysis above. The proposition implies that the rate of growth in the proportion of the population consuming the new substitute good $x_1(t)$ is decreasing in the tax, $\tau_1(t)$, and increasing in the tax, $\tau_2(t)$, over

[7] At $a(t) = 1$, no consumers utilize the old good so the tax revenue from the old commodity is zero as implied by $\tau_2 = 0$. It is clear that $\tau_2 > 0$ would still generate no revenue from the old commodity for $t \geq t'$.

time. If there exists $\tau_1(t)$ and $\tau_2(t)$ such that the efficient ESS where every consumer adopts the new technology good is achieved at time $t = t'$, the learning ceases at time t' and all consumers still present in the population earn the average payoff $\bar{U} = U_1$ for $t \geq t'$. $a(t)$ achieves the upper limit 1 at time t' and $a'(t) = 0$ for $t \geq t'$. Clearly, Proposition 2.3 implies that $\frac{\partial g(\cdot)}{\partial \tau_1(t)} = 0$ and $\frac{\partial g(\cdot)}{\partial \tau_2(t)} = 0$ for $t \geq t'$.

At stage 2, the optimal dynamic commodity tax is determined by maximizing the discounted present value of social welfare satisfying the revenue target of the government and accelerating the growth of the consumption of the new substitute good concurrently. The evolution of consumption from an old to a new commodity is represented by a replicator dynamic equation. The government's problem involving the choice of the optimal dynamic commodity tax is as follows:

$$(2.4) \qquad \max_{\{\tau_1(t),\tau_2(t)\}} \int_0^T e^{-rt}\left\{a(t)[V_1(q_1(t),p_3,w)+a(t)e_1]+(1-a(t))[V_2(q_2(t),p_3,w)+(1-a(t))e_2]\right\}dt$$

$$s.t. \ (1) \ \sum_{i=1}^{2} N_i(t)\tau_i(t)x_i(q_i(t),p_3,w) \geq R(t)$$

$$(2) \ a'(t) = g(a(t),q_1(t),q_2(t),p_3,w)$$

where: $q_i(t) = p_i + \tau_i(t)$ for $i = 1,2$. It is assumed that prices and income are stationary. As shown in Appendix A.2, the optimal commodity taxes are characterized as follows:

$$(2.5) \qquad \tau_1(t)S_{11} = -\theta_1(t)x_1(q_1(t),p_3,w) - \left(\frac{\mu(t)}{\lambda(t)N_1(t)}\right)\frac{\partial g(a(t),q_1(t),q_2(t),p_3,w)}{\partial \tau_1(t)}$$

$$(2.6) \qquad \tau_2(t)S_{22} = -\theta_2(t)x_2(q_2(t),p_3,w) - \left(\frac{\mu(t)}{\lambda(t)N_2(t)}\right)\frac{\partial g(a(t),q_1(t),q_2(t),p_3,w)}{\partial \tau_2(t)}$$

where: $\theta_1(t) = \left[1 - \frac{\alpha_1}{\lambda(t)N} - \tau_1(t)\frac{\partial x_1(q_1(t),p_3,w)}{\partial w}\right]$ and $\theta_2(t) = \left[1 - \frac{\alpha_2}{\lambda(t)N} - \tau_2(t)\frac{\partial x_2(q_2(t),p_3,w)}{\partial w}\right]$ both of which are positive. $\lambda(t) > 0$ is the Lagrangian multiplier associated with the government's budget constraint and $\mu(t) > 0$ is the multiplier associated with the replicator dynamics equation.

The optimal level of $\tau_i(t)$ for $i = 1, 2$ is as follows:

$$\tau_1(t) = \frac{-\theta_1(t)x_1(q_1(t), p_3, w)}{S_{11}} - \left(\frac{1}{S_{11}}\right)\left(\frac{\mu(t)}{\lambda(t)N_1(t)}\right)\frac{\partial g(a(t), q_1(t), q_2(t), p_3, w)}{\partial \tau_1(t)} \tag{2.7}$$

$$\tau_2(t) = \frac{-\theta_2(t)x_2(q_2(t), p_3, w)}{S_{22}} - \left(\frac{1}{S_{22}}\right)\left(\frac{\mu(t)}{\lambda(t)N_2(t)}\right)\frac{\partial g(a(t), q_1(t), q_2(t), p_3, w)}{\partial \tau_2(t)} \tag{2.8}$$

The optimal dynamic commodity tax includes two components. First, the dynamic equivalent to the standard Ramsey Rule. Second, the commodity tax is inversely related to the impact the tax has on the growth rate in the proportion of population adopting the new substitute good $x_1(t)$ over time. Clearly, the optimal dynamic tax rate minimizes both the distortion in consumption and the deceleration in the growth of the consumption of the new substitute good. Hence, Proposition 2.3 implies that when the new substitute good $x_1(t)$ is fully adopted at time t' (i.e., $a(t) = 1$ at $t = t'$), then, $\frac{\partial g(\cdot)}{\partial \tau_1(t)} = \frac{\partial g(\cdot)}{\partial \tau_2(t)} = 0$ for $t \geq t'$. The optimal dynamic commodity tax for good $x_1(t)$ thus collapses to its static counterpart which is determined by the standard Ramsey Rule for $t \geq t'$ and represented by $\tau_1(t')$ given $a(t) = 1$ for $t \geq t'$.

Proposition 2.4. *If an efficient evolutionary stable equilibrium exists at* $t = t'$*, then* $\tau_1(t) < \tau_1(t')$ *and* $\tau_2(t) > \tau_2(t')$ *for* $t < t'$ *and, for* $t \geq t'$, $\tau_1(t) = \tau_1(t')$ *and* $\tau_2(t) = \tau_2(t') = 0$.

Clearly, $\tau_1(t')$ and $\tau_2(t') = 0$ are the optimal commodity tax rates at equilibrium. The proposition can be verified by noting that Proposition 2.3 implies that $\partial g(\cdot)/\partial \tau_1(t) < 0$ and $\partial g(\cdot)/\partial \tau_2(t) > 0$ for $t < t'$ in equations 2.7 and 2.8. Given $S_{ii} < 0$ for $i = 1, 2$, it is easy to verify that for $t < t'$, the optimal dynamic tax rate for new technology, $\tau_1(t)$, is lower than and the optimal dynamic tax rate for old product, $\tau_2(t)$, is higher than their equivalences at equilibrium, $\tau_1(t')$ and $\tau_2(t')$ respectively. On the other hand, eventually, $\tau_1(t)$ and $\tau_2(t)$ will reach the value of $\tau_1(t')$ and $\tau_2(t') = 0$ at time t' since the learning ceases at $t = t'$ and $\partial g(\cdot)/\partial \tau_i(t) = 0$, $i = 1, 2$ for time $t \geq t'$ (Proposition 2.3). The static tax rate for the new substitute good, $\tau_1(t')$, which is determined by the standard Ramsey Rule applies for $t \geq t'$. Proposition 2.4 can be interpreted as the product cycle of the new substitute good x_1, being represented by a standard marketing S-shaped curve. For instance, the introduction of the CD-R or CD-RW involved a product cycle that saw it become a mature product, while co-existing in the market place with the older technology (i.e., the blank audio cassette). The intuition is as follows: The value of the dynamic tax rate for new technology good $x_1(t)$ is much lower when first introduced than when it becomes a mature product so inducing the consumers to adopt the new technology quickly at the beginning of the product cycle. For the same reason, the dynamic tax rate for the old product is set to be higher than its equilibrium equivalence at the beginning. Given $U_1 > U_2$, more and more type 2 consumers would like to learn to utilize the new substitute good, which is represented by $a(t)$ increasing quickly as time proceeds. When the new technology becomes popular on the market, the government chooses to increase the tax rate for good $x_1(t)$ and decrease the tax rate for good $x_2(t)$ slightly with time, but will still keep $a(t)$ arising continuously. When the number of consumers using the new technology reaches a sufficiently large value, the government could increase the tax rate for $x_1(t)$ and reduce the tax rate for $x_2(t)$ in a faster speed, however, the evolution in consumption would continue given $U_1 > U_2$. Eventually, the new substitute good $x_1(t)$ is fully adopted and every consumer gets the equivalent payoff $\bar{U} = U_1$ at time $t = t'$. The economy achieves an efficient evolutionary stable equilibrium where $a(t) = 1$ at time t'. For $t \geq t'$, only new technology goods remain in the market place (i.e., $a(t) = 1$ for

$t \geq t'$), and the static tax rate at equilibrium, $\tau_1(t')$, is imposed on the new commodity for $t \geq t'$. Proposition 2.3 and 2.4 therefore imply the following corollary:

Corollary 2.5. *If Proposition 2.3 and 2.4 hold, then it follows that* $\dfrac{\partial \tau_1(t)}{\partial t} > 0$ *and* $\dfrac{\partial \tau_2(t)}{\partial t} < 0$ *for* $t < t'$.

However, for $t < t'$, $\tau_1(t)$ and $\tau_2(t)$ are potentially not monotonic over time. For instance, at some time t, especially in the periods at the beginning, the value of $\tau_1(t)$ may be lower and the value of $\tau_2(t)$ may be higher, to accelerate growth in the adoption of the new technology as implied by the optimal dynamic commodity taxes in equations 2.7 and 2.8.

Assume the economy achieves the efficient evolutionary stable equilibrium where $a(t) = 1$ under dynamic tax rates at time $t = t'_d$. Equilibrium payoff for the government by the dynamic subgame is defined by $W(\tau_1^*(t'), \tau_2^*(t')) = V_1(p_1 + \tau_1^*(t'), p_3, w) + e_1$.

At stage 1, the government chooses whether to impose a static or dynamic commodity tax scheme. Given $a(t_0) \geq a_3^*(t)$, there should exist both the static and dynamic tax rates making $U_1(t) > U_2(t)$ for $t > t_0$. Therefore, for both the static subgame (i.e., in which the tax rates (τ_1^*, τ_2^*) are imposed at time t_0), and the dynamic subgame, the consumption of the new technology good increases over time, and the social welfare improves over time correspondingly. However, the rate of growth of the type 1 consumers who choose the new technology good should be higher under dynamic tax rates in comparison to the static taxes which minimize the discouragement of consumption. The dynamic taxes minimize the distortion in consumption and the impact of the tax on the growth of the consumption of the new good. As a result, the value of social welfare under dynamic tax rates may be smaller at the beginning but grow with a higher speed, achieving the evolutionary stable equilibrium $t = t'_d$ faster. Given $a(t_0) \geq a_3^*(t)$, if the initial proportion of type 1 consumers in the population at time t_0 was relatively low (i.e., $a(t_0)$ was equal to or only slightly greater than $a_3^*(t)$), the value of $U_1(t) - U_2(t)$ at $t = t_0 + 1$ could be very small under static taxes.

Therefore, the consumption of new commodity grows slowly under static taxes imposed at t_0 especially at the beginning. Notwithstanding, with the same level of $a(t_0)$, the initial dynamic tax rates could be set, resulting in $U_1(t) - U_2(t)$ being larger at $t = t_0 + 1$ and the dynamic taxes from then on could also keep the growth rate of $a(t)$ at a higher level. Consequently, the evolution of the consumption of the new technology good could be quicker, and the improvement in the social welfare higher, under dynamic taxes compared to static taxes, when the value of $a(t_0)$ is relatively small. In contrast, if the proportion of consumers purchasing the new technology good is sufficiently large at time t_0 (i.e., $a(t_0)$ was largely greater than $a_3^*(t)$), the difference between $U_1(t)$ and $U_2(t)$ could be large at time $t = t_0 + 1$ even under static tax rates. Additionally, the value of $U_1(t) - U_2(t)$ increases with the growth in $a(t)$ over time. Hence, with static tax rates, the proportion of type 1 consumers raises quickly and the level of social welfare increases with a high speed as well. Therefore, the dynamic paths of consumption of the new good and the improvement in social welfare under static and dynamic taxes become very similar when $a(t_0)$ is sufficiently large. For simplicity, the administrative adjustment cost for each time period t when a dynamic tax policy is implemented is assumed to be zero. The government may consider implementing a dynamic tax policy, if the level of aggregate social welfare under dynamic tax rates is higher as follows:

$$(2.9)\ \int_0^{t'_d} e^{-rt}\left\{a_d(t)\left[V_1(p_1+\tau_1^*(t),p_3,w)+a_d(t)e_1\right]+(1-a_d(t))\left[V_2(p_2+\tau_2^*(t),p_3,w)+(1-a_d(t))e_2\right]\right\}dt + \int_{t'_d}^{T} e^{-rt}\left[V_1(p_1+\tau_1^*(t'),p_3,w)+e_1\right]dt \geq \int_0^{t'_s} e^{-rt}\left\{a_s(t)\left[V_1(p_1+\tau_1^*,p_3,w)+a_s(t)e_1\right]+(1-a_s(t))\left[V_2(p_2+\tau_2^*,p_3,w)+(1-a_s(t))e_2\right]\right\}dt + \int_{t'_s}^{T} e^{-rt}\left[V_1(p_1+\tau_1^*(t'),p_3,w)+e_1\right]dt$$

Clearly, equation 2.9 would be more likely to hold when the adjustment period for the new commodity to become a mature product under dynamic tax rates is sufficiently shorter than that under static taxes (i.e., $t'_d < t'_s$ sufficiently). As shown above, the evolution of the new technology good is much quicker and thus the improvement in social welfare is higher under dynamic tax rates when initial proportion of type 1 consumers in the population is small. As a result, given the government policy time horizon T, the imposition of a dynamic commodity tax can be recommended when the initial proportion of consumers adopting new technology good is sufficiently small. While, when the type 1 consumers in the population purchasing the new good is a sufficiently large value initially, a static tax policy can be recommended to the government instead of the dynamic tax policy. Additionally, the revenue target of the government is reached for each period in the dynamic subgame (i.e., $\sum_{i=1}^{2} N_i(t)\tau_i(t)x_i(q_i(t), p_3, w) = R(t)$ for $t \in [t_0, T]$). Whereas under static tax rates, the tax revenue constraint of the government, $\sum_{i=1}^{2} N_i\tau_i x_i(q_i, p_3, w) = R(t)$, is satisfied for $t = t_0$ and $t \in [t'_s, T]$. For the time between t_0 and t'_s, the revenue constraint of the government is not binding for the static tax rates. It is associated with the fact that given $a(t_0) \geq a_3^*(t)$, proportion of type 1 consumers grows for $t_0 < t < t'_s$ (i.e., $a'(t) > 0$ for $t_0 < t < t'_s$). However, under static tax rates, the demands for the new or old commodity by each consumer, $x_i(q_i, p_3, w)$, $i = 1, 2$, are not changed at the same after tax prices $p_1 + \tau_1$ and $p_2 + \tau_2$. The values of $\tau_i x_i(q_i, p_3, w)$, $i = 1, 2$, remain the same for $t \in (t_0, t'_s)$. Consequently, during the adjustment period for the new commodity to become mature, the revenue target is not satisfied any more with the growth in $a(t)$ for $t \in (t_0, t'_s)$. The quantity demanded by each type 1 consumer is less than the quantity demanded by each type 2 consumer due to the higher price charged for the new technology good. Moreover, the levies placed on the new commodity can not be much higher than that on the old product before the new good is fully adopted. As a result, generally, $\tau_1 x_1(q_1, p_3, w) < \tau_2 x_2(q_2, p_3, w)$. With the growth in $a(t)$ over time (i.e., the growth in N_1), $\sum_{i=1}^{2} N_i \tau_i x_i(q_i, p_3, w)$, $i = 1, 2$, would be more and more less than the revenue target of the government $R(t)$ which implies that the government receives less tax revenue by imposing the static commodity tax scheme. Additionally, the longer the adjustment period for the new product to become mature under static taxes, the more revenue loss by the government. Consequently, when the level of $a(t_0)$ is sufficiently low (i.e., the adjustment period for the new product is much shorter under dynamic taxes), imposing the dynamic tax policy not only benefits the consumers but also makes the government being better off. Given the government revenue is higher under dynamic taxes, the decision of government's commodity tax policy may also depend on whether the industry receives a share of the government revenue from commodity taxes. For instance, for the CD and cassette industry, the levies imposed by the government are going to be used to compensate the revenue lost of music copyright owners. Therefore, with the revenue sharing, the government may consider implementing the dynamic tax policy even if the total social welfare is higher under static taxes. The inability to have the revenue constraint binding throughout the entire adjustment period of the new commodity to become mature when a static taxation scheme is imposed makes welfare analysis complicated. In what follows, the empirical analysis provides the basis for further comparisons of the welfare implication between the static and dynamic commodity taxes, and interpretation of the results.

2.4 EMPIRICAL APPLICATION

The theoretical model implies that the optimal dynamic taxation rates will be defined by equations 2.7 and 2.8. An empirical model is developed to explore the determination of these values using examples of new product introduction along with dynamic commodity taxation. The two examples utilized include: first, new technology TVs (LCD TVs and Plasma TVs) and old technology TVs (Tube TVs); second, blank CD-Rs and blank audio cassettes used for recording and listening to music. Consumer preferences are represented by CES utility functions. The values used in the analysis are shown in Table 2.1.[8]

The optimal control problem defined by equation 2.4 was solved by collocation method in which the value function is approximated by a linear combination of some known basis functions over the space of state variable. Hence, the infinite dimensional optimization problem in continuous control variables is transformed into a finite dimensional nonlinear programming (NLP) problem which can be solved by any gradient-based method (e.g., a SQP method). The original continuous aggregate social welfare is thus approximated by a linear combination of the social welfare functions at some specified collocation points for state variable $a(t)$. At collocation point i, the value of collocation function, W_i, is obtained by solving the optimization problem defined by equation 2.4 given the value of $a(t)$ at point i. Therefore, the optimal dynamic tax rates for each time interval are those that both maximize the value of current welfare and maximize the increase in total welfare associated with the growth in $a(t)$. In our two specific examples, in order to test the predictions implied by the theoretical model, the value of $a(t) = a_u$ making $U_1(t) = U_2(t)$ is first found and the initial value of $a(t)$ at $t = t_0 = 0$ is set to be greater than a_u. The positions of the collocation points are determined by a sequential way. For instance, $a(t+1)$ is determined with the values of $a(t)$ and $a'(t)$ using first-order Taylor approximations. Furthermore, the revenue targets of the government at each time interval under dynamic tax rates are fixed at the initial level.

[8]The prices for LCD TV and Tube TV are the current prices at Future-shop Inc., June 2007, and the prices for the blank CD-R and blank audio cassette were the prices of 2001 at Future-shop Inc. Ratio of externality for LCD TV and Tube TV is generated by comparing the digital video resolutions for both TVs. (Source: en.wikipedia.org/wiki/1080i) and that for CD and cassette is determined by the number of songs could be recorded by the two media. The revenue target under dynamic environment is supposed to be fixed at the level at the beginning. For example 1, the revenue target is set as 8% of the total expenditure on TV and, for example 2, the revenue target is equal to the revenue of levies on blank CD-R and blank audio cassette for year of 2001 using the data from Canadian Copy-right Board. The annual income for the consumer is generated as the average expenditure per person and the expenditure share is defined as the ratio of the expenditure on the concerned products to the total expense using the data of year of 2001. (Source: Spending Pattern in 2001, Statistics Canada).

The empirical results for the LCD TV/Tube TV case are shown in Table 2.2 to Table 2.5. The results for the blank CD-R/blank audio cassette case are shown in Table 2.6 to Table 2.9. Figure 2.3 to Figure 2.6 show the dynamics over time of optimal commodity taxation rates, the evolution of consumption of the new technology TVs, and improvement over time of social welfare under static versus dynamic taxes when initial proportion of consumers purchasing new LCD TV is relatively low at 0.192. Comparably, Figure 2.7 to 2.10 show the same dynamic paths when the initial proportion of type 1 consumers raises to 0.3. In addition, for the CD-R/audio cassette case, Figures 2.11 to 2.12 and Figures 2.13 to 2.14 draw the graphs for the evolution in $a(t)$ and improvement in $W(t)$ under dynamic versus static tax rates, when the initial value of $a(t)$ is 0.14 and 0.25 respectively.

2.4.1 EMPIRICAL OPTIMAL COMMODITY TAXATION RATES

In the first example of new technology and old technology TVs, the price for the new LCD TV is much higher than that of the old tube TV: $p_1 > p_2$. The value of $a(t)$ making $U_1(t) = U_2(t)$ is found by maximizing the welfare function satisfying the revenue target and the condition of $U_1(t) = U_2(t)$. The resulting value of $a(t) = a_u$ is equal to 0.1918. Thus, we first set the initial value of proportion of type 1 consumers to $a(t_0) = 0.192$ which is slightly greater than 0.1918. Next, we raise the level of $a(t_0)$ to be equal to 0.3 and compare the results.

When $a(t_0) = 0.192$, the static tax rate for the new technology TV is almost as high as that for the old tube TV ($\tau_1 = 108.6262$ and $\tau_2 = 112.9292$). As a result, the rate of growth of consumption of new commodity is low at the beginning in the static framework (i.e., $a_s'(t_0) = 1.7653e-005$). In contrast, the dynamic tax rate for new technology TV at initial time $t_0 = 0$ is much lower than the tax rate under static environment ($\tau_1(t_0) = 48.5548 < \tau_1 = 108.6262$), which makes $V_1(t_0)$ higher, while the dynamic tax rate for the old tube TV at time t_0 is higher than its static counterpart ($\tau_2(t_0) = 118.0403 > \tau_2 = 112.9292$) making $V_2(t_0)$ lower. Consequently, under dynamic tax rates, the value of $U_1(t_0) - U_2(t_0)$ is higher, so type 2 consumers will deviate to purchase new product at a quicker speed under the dynamic environment($a'_d(t_0) = 0.0021$). However, the dynamic tax rate provides a lower level of social welfare at the beginning compared with the static tax rate ($W_d(t_0) = 220.9609 < W_s(t_0) = 220.9614$), which is due to the dynamic tax rate not only maximizing social welfare but also ensuring the growth in consumption of the new technology good. The proportion of type 1 consumers grows slowly under static tax rates for the first a few periods. Starting from period 7, the growth rate of $a(t)$ becomes higher and hence $a(t)$ exceeds the value of 0.3 by period 13. From then on, $a(t)$ grows very quickly reaching the equilibrium value of 1 at $t = t_{14}$. Correspondingly, social welfare $W(t)$ raises slightly for the first few periods, and then grows with faster reaching the value of 225.7624 at equilibrium at $t = t_{14}$ (i.e., $t'_s = 14$). Comparatively, under dynamic tax rates, the consumption of the new technology TV grows at a higher speed starting from period 1. As a result, $a(t)$ exceeds the value of 0.3 by period 7, reaching the upper limit value of 1 at time $t = t_8$ (i.e., $t'_d = 8$). The level of social welfare under dynamic taxes is lower than its static counterpart initially, but rises with a higher speed with the growth in $a(t)$. $W(t)$ reaches the equilibrium value of 225.7624 at $t = t_8$ for the dynamic subgame. For $t \geq t_8$, $W(t)$ remains constant at 225.7624. Since $t'_d < t'_s$ and the difference between $W_s(t_0)$ and $W_d(t_0)$ is not large, the value of aggregate social welfare seems higher in the dynamic framework. Thus, when number of consumers purchasing new technology TV is relatively low at the beginning (i.e., $a(t_0) = 0.192$), dynamic commodity tax scheme can be recommended to the government which is consistent with the result implied by the theoretical model. Table 2.2 shows that dynamic tax rate for new LCD TV increases slowly and dynamic tax rate for old Tube TV declines slightly for the first a few period. Then, $\tau_1(t)$ raises faster and faster and $\tau_2(t)$ continues to decrease slightly with time but keeping the growth in $a(t)$ high. When the value of $a(t)$ exceeds 0.5, $\tau_1(t)$ increases sharply and $\tau_2(t)$ declines more steeply. The static tax rates at equilibrium $\tau_1'(t) = 417.4586$ and $\tau_2'(t) = 0$ then apply for $t \geq t_8$. Clearly, the movement over time of dynamic tax rates is in alignment with the predictions implied by the theoretical framework. Table 2.2 also shows that revenue target of the government is not satisfied for $t \in (t_0, t_{14})$ under static tax rates. The commodity tax revenue decreases with the growth in $a(t)$. The budget constraint is binding again for $t \geq t_{14}$ when tax rates at equilibrium $\tau_1'(t) = 417.4586$ and $\tau_2'(t) = 0$ apply. [9] Consequently, government receives more tax revenue by implementing a dynamic tax policy.

When the initial proportion of type 1 consumers purchasing new technology TV is raised to be 0.3 (i.e., $a(t_0) = 0.3$), the static tax rate for LCD TV is much higher than its counterpart when $a(t_0) = 0.192$ ($\tau_1 = 285.9766$). Additionally, the tax rate for Tube TV is lower when the level of $a(t_0)$ is higher ($\tau_2 = 106.5183$). Comparatively, the initial dynamic tax rates at $t = t_0$ are $\tau_1(t) = 202.0584$ and $\tau_2(t) = 116.7546$ which are lower and higher than their static counterparts respectively. However, with the high level of $a(t_0)$, the growth rate of consumption of new commodity is high at the beginning even under the static tax rates (i.e., $a_s'(t_0) = 0.1521$). Additionally, $a(t)$ increases with time more rapidly, achieving the equilibrium value of 1 at time $t = t_3$ under static taxes (i.e., $t_s' = 3$). While under the dynamic tax rates, $a(t)$ raises with a slightly higher speed, also reaching the value of 1 at the beginning of the fourth period $t = t_3$ (i.e., $t_d' = 3$). Figure 2.9 and Figure 2.10 show that the evolution of consumption of new technology TV and the improvement in the social welfare over time are almost equivalent for the static and dynamic subgames. Therefore, the government will consider to impose a static tax scheme when the initial proportion of type 1 consumers in the population is sufficiently large. However, Table 2.4 shows that

[9] For $t \geq t'$, $\tau_2(t)$ may still be positive. However, there is no welfare or revenue implications since $1 - a(t) = 0$ for $t \geq t'$. Hence, for simplicity, $\tau_2(t)$ is considered to be zero for $t \geq t'$.

revenue of government is declining over time under static taxes for $t \in (t_0, t_3)$. Therefore, the government may be recommended to adopt the dynamic as opposed to the static tax policy if the industry shares the commodity tax revenue of the government.

The second example is for the blank CD-R and blank audio cassette where the price for the new technology good is lower than the price for the old product: $p_1 < p_2$ (see Table2.1). In this case, the value of $a(t)$ making $U_1(t) = U_2(t)$ is found to be equal to $a_u = 0.1374$ which is smaller than the value of a_u in the TV case. This may be associated with the fact that the price for the new substitute good CD-R is lower than the price for the old product audio cassette in this case. We thus first set the value of initial proportion of consumers adopting new CD-R at $t = t_0$ as $a(t_0) = 0.14$ which is slightly greater than 0.1374. Then, we increase the value of $a(t_0)$ to 0.25 to compare the results.

When the number of type 1 consumers is small initially (i.e., $a(t_0) = 0.14$), the static tax rate for the new substitute CD-R is $\tau_1 = 0.28223$ and the static tax rate for the old product cassette is $\tau_2 = 0.74874$. Correspondingly, the rate of growth of consumption of new commodity is extremely low at the beginning in the static framework (i.e., $a'_s(t_0) = 1.1081e-009$). It is associated with the factors that the proportion of consumers purchasing new good CD-R is low and the levies imposed on CD-R is high. Comparatively, the dynamic tax rate for new product CD-R at initial time $t = t_0$ is much lower than the tax rate under static environment ($\tau_1(t_0) = 0.000324 < \tau_1 = 0.28223$) which makes $V_1(t_0)$ being much higher while the dynamic tax rate for the old commodity cassette at time t_0 is higher than its static counterpart ($\tau_2(t_0) = 0.91267 > \tau_2 = 0.74874$) making $V_2(t_0)$ be largely lowered.[10] Consequently, even with a relatively small initial value of $a(t)$, under dynamic tax rates, the growth rate of consumption of new technology good is not low at the beginning ($a'_d(t_0) = 0.00803$). However, the dynamic tax rate provides a lower level of social welfare

[10]It is useful to note that the actual levy rate on CD-R was 5.2 cents in 1999 and 21 cents by 2003.

at time $t = t_0$ compared with that provided by the static tax rate ($W_d(t_0) = 2244.0786 < W_s(t_0) = 2244.0809$). The proportion of type 1 consumers grows extremely slowly under static tax rates. $a'(t)$ does increase with time but the increase in $a'(t)$ is very small. As a result, $a(t)$ only reaches the value of 0.140081 by the end of the policy time horizon $t = t_{20} = T$. The social welfare raises very slowly as well with the little growth in $a(t)$, only reaching the value of 2244.08089 at time $t = T$. Thus, compared with the TV case, the social welfare is not improved much in the static framework for the CD/Cassette case, which is mainly due to the low initial value of $a(t)$. In contrast, under dynamic tax rates, the consumption of new technology good CD-R grows fast starting from period 1. The growth rate of $a(t)$ increases over time and becomes much higher when $a(t)$ exceeds the value of 0.25. As a result, proportion of type 1 consumers reaches the equilibrium value of 1 at time $t = t_7$ (i.e., $t'_d = 7$). Correspondingly, the level of social welfare under dynamic taxes also arises quickly with the growth in $a(t)$. $W(t)$ reaches the equilibrium value of 2248.3732 at $t = t_7$ for the dynamic subgame. For $t \geq t_7$, $W(t)$ remains constant at 2248.3732. Apparently, the value of aggregate social welfare is higher in the dynamic framework. Thus, dynamic commodity tax scheme can be recommended to the government when the number of consumers purchasing new substitute good CD-R is sufficiently low at the beginning (i.e., $a(t_0) = 0.14$). Table 2.6 shows that dynamic tax rate for CD-R increases over time and dynamic tax rate for audio cassette declines over time, reaching the values at equilibrium at time $t = t_7$. It is also shown by Table 2.6 that the commodity tax revenue of government decreases with the growth in $a(t)$ for $t \in (t_0, T)$ under static tax rates.

When the initial proportion of type 1 consumers purchasing new technology good CD-R is raised to be 0.25 (i.e., $a(t_0) = 0.25$), the static tax rate for CD-R is higher than its counterpart when $a(t_0) = 0.14$ ($\tau_1 = 0.3057$). Additionally, the static tax rate for audio cassette is lower when the level of $a(t_0)$ is higher ($\tau_2 = 0.7483$). Comparatively, the initial dynamic tax rates at $t = t_0$ are $\tau_1(t) = 0.2461$ and $\tau_2(t) = 0.8003$ which are lower and higher than their static counterparts respectively. Unlikely the case in which $a(t_0)$ is small, with the high level of $a(t_0)$, the growth rate of consumption of new technology good is high at the beginning even under the static tax rates (i.e., $a'_s(t_0) = 0.123$). Additionally, $a(t)$ increases with time quicker and quicker, achieving the equilibrium value of 1 at the beginning of the fourth period $t = t_3$ under static taxes (i.e., $t'_s = 3$). While under dynamic tax rates, $a(t)$ raises with a slightly higher speed, also reaching the value of 1 at time $t = t_3$ (i.e., $t'_d = 3$). Figure 2.13 and Figure 2.14 show that the evolution of consumption of new commodity CD-R and the improvement in the social welfare over time are equivalent for the static and dynamic subgames. Therefore, the government will consider to impose a static tax scheme when the initial proportion of type 1 consumers in the population is sufficiently large. However, Table 2.8 shows that revenue of government is declining over time under static taxes for $t \in (t_0, t_3)$. Therefore,, the government may be recommended to adopt the dynamic as opposed to the static tax policy if the industry shares the commodity tax revenue of the government. Clearly, all results for the CD-Cassette case when $a(t_0) = 0.25$ are similar with those for the TV case when $a(t_0) = 0.3$.

In both examples, the whole picture of dynamics in optimal commodity tax rates over time could be drawn roughly. The empirical results show that the dynamic tax rate for the new technology good is set much lower whereas the dynamic tax rate for the old product is set much higher than their static counterparts respectively at the beginning given the equivalent initial value of $a(t)$ at $t = t_0$. The initial dynamic tax rates make the new substitute good more attractive, so the proportion of type 1 consumers increases with time faster. Then, when the new technology good becomes more popular on the market, the government can consider gradually increasing the tax rate $\tau_1(t)$ and decreasing the tax rate $\tau_2(t)$, while keeping the level of $a'(t)$ rising. When the number of consumers utilizing the new product becomes sufficiently large, the government has the option of speeding up the increase in $\tau_1(t)$ and the decrease in $\tau_2(t)$ further. Eventually, all consumers adopt the new technology good at some time $t = t'$ and the economy achieves an evolutionary stable equilibrium at t'. For $t \geq t'$, the static tax rate determined by the standard Ramsey Rule applies. In addition, the empirical results also show that the growth in consumption of the new technology good is much faster and the improvement in social welfare is more under dynamic taxes when the initial proportion of consumers adopting new commodity is sufficiently small. Further, government revenue of commodity taxes decreases with $a(t)$ by imposing static tax scheme during the adjustment period of the new technology good to become fully adopted. Apparently, the empirical results in both examples are consistent with the predictions implied by the theoretical framework.

2.4.2 WELFARE COMPARISON AND POLICY CONSIDERATIONS

Figures 2.6 and 2.10 and Figures 2.12 and 2.14 show the welfare improvements under static v.s dynamic tax rates with different initial values of $a(t)$ for the LCD/Tube TV case and for the CD-R/Audio Cassette case respectively. Clearly, in both cases, the level of social welfare is lower under dynamic tax rates at the beginning. It is attributed to the reason that compared with the static tax rates, the dynamic tax rates not only maximizes the social welfare but also accelerates the consumption of the new technology good at the meantime. Figures show that when the proportion of consumers purchasing new technology good is relatively small at the beginning, consumption of the new commodity grows with a much higher speed under dynamic taxes, therefore, the level of social welfare raises faster as well. However, when the initial number of type 1 consumers is sufficiently large, evolution of consumption of the new technology good becomes fast under the static tax rates as well. As a result, the improvements in social welfare are almost equivalent for both the static and dynamic framework. This implies that government would be more likely to be recommended to impose a dynamic commodity tax scheme when the initial value of $a(t)$ is sufficiently small. Additionally, the government receives less revenues under static taxes. Thus, the government may consider to implement the dynamic as opposed to the static tax policy even if the latter provides a higher level of total social welfare when the industry has the share of government revenue.

2.5 CONCLUSION

The paper characterizes the optimal commodity taxation scheme for an economy with an evolutionary process in the adoption of a new substitute good. The adaptation of the new good is represented by a replicator dynamic equation and referred to as consumption replicator dynamics. The government chooses the commodity taxes in order to maximize a social welfare function, subject to a revenue constraint and a replicator dynamic equation. The replicator dynamic equation, as in Samuelson (1998) and Hauert *et al* (2002), represents the evolution in the proportion of the population consuming the new substitute good, as opposed to the old good in a three good economy. The new and old goods are used in household production and may be considered to be new LCD television and old tube TV, respectively. Households are assumed to possess a home production technology that uses either the old or the new good, but not both. Households also care about a composite good.

The key findings of the study are as follows: First, the paper finds that the optimal commodity tax, in the presence of consumption replicator dynamics, involves the taxes minimizing the marginal deadweight loss or distortion in consumption as asserted by the standard Ramsey Rule, and minimizing the impact on the growth in the consumption of the newly introduced substitute commodity. Second, the welfare improvement from implementing the dynamic as opposed to the static commodity tax is high when the initial proportion of type 1 consumers in the population is sufficiently low. Therefore, the government chooses to implement a dynamic commodity tax scheme over a static commodity tax policy, when the number of consumers who adopt new technology commodity are sufficiently small at the beginning. Additionally, the government revenue of commodity taxes decreases over time under static tax rates during the period over which the new commodity adjust from being a initially introduced to a mature good. As a result, if the industry shares the government revenue, the dynamic tax scheme may be recommended even if the aggregate social welfare is higher under static taxes. The empirical results of the optimal commodity taxation rates for both the LCD TV/Tube TV case and the CD-R/Audio Cassette case are consistent with the predictions of the theoretical framework.

This paper contributes to the literature on optimal commodity taxation through the application of the concepts of evolutionary game theory to the commodity tax literature. The bargaining activity between various players in the tax policy framework clearly represents scope for further study. Furthermore, the use of dynamic commodity taxation as part of environment policy, such as promoting the purchase of hybrid engine cars over combustion engines, could be studied within the context of the framework presented in this paper.

Table 2.1: Data used in analysis for both LCD TV/Tube TV case and CD-R/Cassette case

Variable	LCD TV and Tube TV		Blank CD-R and Blank Audio Cassette	
	LCD TV	Tube TV	Blank CD-R	Blank Audio Cassette
Price(p_1, p_2)	1571.4	422.2	1	2
Price for composite goods(p_3)	100	100	10	10
Externality(e_1, e_2)	6	1	5	1
Income/Expenditure(w)	22459	22459	22459	22459
Expenditure share(α)	0.0033	0.0033	0.0002	0.0002
Tax revenue target (R(t)=R)	177,875,280	177,875,280	16,172,000	16,172,000
Total population(N)	3.0e+007	3.0e+007	3.0e+007	3.0e+007
Discount factor($\rho = r$)	0.1	0.1	0.1	0.1

Source: Future-Shop Inc., en.wikipedia.org/wiki/1080i and Statistics Canada

Table 2.2: Movement over Time of Related Variables under Static vs. Dynamic Taxes

–LCD/Tube TV Case ($a_{t_0} = 0.192$)

Static Subgame										
t	τ_1	τ_2	a(t)	V_1	V_2	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t∈ $[t_0, t_1)$	108.6262	112.9292	0.192	219.8095	220.1534	220.9615	220.9614	1.7653e-005	220.9614	177875280
t∈ $[t_1, t_2)$	108.6262	112.9292	0.19202	219.8095	220.1534	220.9616	220.9614	3.6826e-005	220.96141	177872587
t∈ $[t_2, t_3)$	108.6262	112.9292	0.19205	219.8095	220.1534	220.9618	220.9613	7.6831e-005	220.96142	177867024
t∈ $[t_3, t_4)$	108.6262	112.9292	0.19213	219.8095	220.1534	220.9623	220.9612	1.603e-004	220.96145	177855419
t∈ $[t_4, t_5)$	108.6262	112.9292	0.19229	219.8095	220.1534	220.9632	220.9611	3.347e-004	220.9615	177831201
t∈ $[t_5, t_6)$	108.6262	112.9292	0.19263	219.8095	220.1534	220.9653	220.9608	6.996e-004	220.9616	177780637
t∈ $[t_6, t_7)$	108.6262	112.9292	0.19333	219.8095	220.1534	220.9694	220.96	0.001465	220.9619	177674961
t∈ $[t_7, t_8)$	108.6262	112.9292	0.19479	219.8095	220.1534	220.9782	220.9586	0.00308	220.9624	177453632
t∈ $[t_8, t_9)$	108.6262	112.9292	0.19787	219.8095	220.1534	220.9967	220.9555	0.0065	220.9637	176988024
t∈ $[t_9, t_{10})$	108.6262	112.9292	0.2044	219.8095	220.1534	221.036	220.949	0.0142	220.9668	175999550
t∈ $[t_{10}, t_{11})$	108.6262	112.9292	0.2186	219.8095	220.1534	221.121	220.9348	0.0318	220.9755	173861419
t∈ $[t_{11}, t_{12})$	108.6262	112.9292	0.2504	219.8095	220.1534	221.3117	220.903	0.0767	221.0053	169059573
t∈ $[t_{12}, t_{13})$	108.6262	112.9292	0.3271	219.8095	220.1534	221.7719	220.8263	0.2081	221.1356	157474561
t∈ $[t_{13}, t_{14})$	108.6262	112.9292	0.5352	219.8095	220.1534	223.0205	220.6182	0.5976	221.9038	126040554
t∈ $[t_{14}, T]$	417.4586	0	1	219.7624	·	225.7624	·	0	225.7624	177875280
Dynamic Subgame										
t	$\tau_1(t)$	$\tau_2(t)$	a(t)	$V_1(t)$	$V_2(t)$	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t∈ $[t_0, t_1)$	48.5548	118.0403	0.192	219.8198	220.1503	220.9718	220.9583	0.0021	220.9609	177875280
t∈ $[t_1, t_2)$	56.9123	117.6432	0.1941	219.8183	220.1506	220.9829	220.9565	0.0041	220.9616	177875280
t∈ $[t_2, t_3)$	76.8257	116.5205	0.1982	219.8149	220.1512	221.0041	220.953	0.0081	220.9632	177875280
t∈ $[t_3, t_4)$	96.945	115.9294	0.2063	219.8115	220.1516	221.0493	220.9453	0.017	220.9667	177875280
t∈ $[t_4, t_5)$	127.3458	115.475	0.2233	219.8064	220.1519	221.1462	220.9286	0.0377	220.9772	177875280
t∈ $[t_5, t_6)$	178.4085	114.9832	0.261	219.7981	220.1522	221.3641	220.8912	0.0912	221.0146	177875280
t∈ $[t_6, t_7)$	261.5603	113.9026	0.3522	219.7851	220.1528	221.8983	220.8006	0.2504	221.1872	177875280
t∈ $[t_7, t_8)$	394.0954	100.9254	0.6026	219.7657	220.1607	223.3813	220.5581	0.6761	222.2593	177875280
t∈ $[t_8, T]$	417.4586	0	1	219.7624	·	225.7624	·	0	225.7624	177875280

Table 2.3: Percentage Changes of Related Variables Over Time under Static vs. Dynamic Taxes

–LCD/Tube TV Case ($a_{t_0} = 0.192$)

Static Subgame							
t	$V_1(t)$(%)	$V_2(t)$(%)	$U_1(t)$(%)	$U_2(t)$(%)	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	0	0	4.5257e-005	0	9.375e-003	4.5257e-006	-0.0056218
$t_1 - t_2$	0	0	9.0513e-005	-4.5257e-005	0.01667	4.5257e-006	-0.0056219
$t_2 - t_3$	0	0	2.2628e-004	-4.5257e-005	0.041656	1.3577e-005	-0.005622
$t_3 - t_4$	0	0	4.0731e-004	-4.5257e-005	0.083277	2.2628e-005	-0.011245
$t_4 - t_5$	0	0	9.5038e-004	-1.3577e-004	0.18	4.5257e-005	-0.02812
$t_5 - t_6$	0	0	1.8555e-003	-3.6206e-004	0.36	1.3577e-004	-0.061874
$t_6 - t_7$	0	0	3.9825e-003	-6.336e-004	0.76	2.2628e-004	-0.123825
$t_7 - t_8$	0	0	8.3719e-003	-1.403e-003	1.58	5.8834e-004	-0.2592
$t_8 - t_9$	0	0	0.017783	-2.9418e-003	3.3	0.0014	-0.55935
$t_9 - t_{10}$	0	0	0.038455	-6.4268e-003	6.95	0.0039	-1.2159
$t_{10} - t_{11}$	0	0	0.086242	-0.014393	14.55	0.0135	-2.76084
$t_{11} - t_{12}$	0	0	0.21	-0.034721	30.63	0.059	-6.8556
$t_{12} - t_{13}$	0	0	0.56	-0.094237	63.62	0.3474	-19.9594
$t_{13} - t_{14}$	-2.1428e-004	·	1.23	·	86.85	1.7389	41.1219
$t_{14} - T$	0	·	0	·	0	0	0
Dynamic Subgame							
t	$V_1(t)$(%)	$V_2(t)$(%)	$U_1(t)$(%)	$U_2(t)$(%)	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	-0.00068	0.00013	0.00502	-0.00081	1.09375	0.00031	0
$t_1 - t_2$	-0.00154	0.00027	0.00959	-0.00158	2.11231	0.00072	0
$t_2 - t_3$	-0.00154	0.00018	0.02045	-0.00348	4.08678	0.00158	0
$t_3 - t_4$	-0.00232	0.00013	0.04383	-0.00755	8.24042	0.00475	0
$t_4 - t_5$	-0.00377	0.00013	0.09853	-0.01692	16.88311	0.01692	0
$t_5 - t_6$	-0.00591	0.00027	0.24132	-0.04101	34.94252	0.07809	0
$t_6 - t_7$	-0.00882	0.00358	0.66832	-0.10982	71.09596	0.4847	0
$t_7 - t_8$	-0.0015	·	1.06593	·	65.94756	1.57613	0
$t_8 - T$	0	·	0	·	0	0	0

Table 2.4: Movement over Time of Related Variables under Static vs. Dynamic Taxes

–LCD/Tube TV Case ($a_{t_0} = 0.3$)

Static Subgame										
t	τ_1	τ_2	a(t)	V_1	V_2	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t$\in [t_0, t_1)$	285.9766	106.5183	0.3	219.7814	220.1572	221.5814	220.8572	0.1521	221.0745	177875280
t$\in [t_1, t_2)$	285.9766	106.5183	0.4521	219.7814	220.1572	222.494	220.7051	0.4431	221.5139	167792481
t$\in [t_2, t_3)$	285.9766	106.5183	0.8952	219.7814	220.1572	225.1526	220.262	0.4588	224.6401	138418674
t$\in [t_3, T)$	417.4586	0	1	219.7624	·	225.7624	·	0	225.7624	177875280
Dynamic Subgame										
t	$\tau_1(t)$	$\tau_2(t)$	a(t)	$V_1(t)$	$V_2(t)$	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t$\in [t_0, t_1)$	202.0584	116.7546	0.3	219.7943	220.1511	221.5943	220.8511	0.1561	221.0741	177875280
t$\in [t_1, t_2)$	303.4176	116.0847	0.4561	219.7788	220.1515	222.5154	220.6954	0.4515	221.5255	177875280
t$\in [t_2, t_3)$	411.8083	105.3448	0.9076	219.7632	220.158	225.2088	220.2504	0.4158	224.7506	177875280
t$\in [t_3, T)$	417.4586	0	1	219.7624	·	225.7624	·	0	225.7624	177875280

Table 2.5: Percentage Changes of Related Variables Over Time under Static vs. Dynamic Taxes

–LCD/Tube TV Case ($a_{t_0} = 0.3$)

Static Subgame							
t	$V_1(t)$(%)	$V_2(t)$(%)	$U_1(t)$(%)	$U_2(t)$(%)	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	0	0	0.41186	-0.06887	50.7	0.19876	-5.66847
$t_1 - t_2$	0	0	1.19491	-0.20077	98	1.41129	-17.506
$t_2 - t_3$	-0.00864	·	0.27084	·	11.70688	0.49960	28.50526
$t_3 - T$	0	·	0	·	0	0	0
Dynamic Subgame							
t	$V_1(t)$(%)	$V_2(t)$(%)	$U_1(t)$(%)	$U_2(t)$(%)	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	-0.00705	1.81693e-004	0.41567	-0.07050	52.03	0.20418	0
$t_1 - t_2$	-0.0071	0.00295	1.21043	-0.20164	98.99	1.45586	0
$t_2 - t_3$	-3.64028	·	0.24582	·	10.1807	0.45019	0
$t_3 - T$	0	·	0	·	0	0	0

Table 2.6: Movement over Time of Related Variables under Static v.s Dynamic Taxes

–CD/Cassette Case ($a_{t_0} = 0.14$)

Static Subgame										
t	τ_1	τ_2	a(t)	V_1	V_2	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t∈ $[t_0, t_1)$	0.2822	0.7487	0.14	2243.38087	2243.22087	2244.08087	2244.08087	1.1081e-009	2244.08087	16171988
t∈ $[t_1, t_2)$	0.2822	0.7487	0.1400000011	2243.38087	2243.22087	2244.08087	2244.08087	1.9086e-009	2244.08087	16171988
t∈ $[t_2, t_3)$	0.2822	0.7487	0.140000003	2243.38087	2243.22087	2244.08087	2244.08087	3.2874e-009	2244.08087	16171988
t∈ $[t_3, t_4)$	0.2822	0.7487	0.1400000063	2243.38087	2243.22087	2244.08087	2244.08087	5.6623e-009	2244.08087	16171988
t∈ $[t_4, t_5)$	0.2822	0.7487	0.140000012	2243.38087	2243.22087	2244.08087	2244.08087	9.7528e-009	2244.08087	16171988
t∈ $[t_5, t_6)$	0.2822	0.7487	0.1400000217	2243.38087	2243.22087	2244.08087	2244.08087	1.6798e-008	2244.08087	16171988
t∈ $[t_6, t_7)$	0.2822	0.7487	0.1400000385	2243.38087	2243.22087	2244.08087	2244.08087	2.8933e-008	2244.08087	16171988
t∈ $[t_7, t_8)$	0.2822	0.7487	0.1400000675	2243.38087	2243.22087	2244.08087	2244.08087	4.9835e-009	2244.08087	16171988
t∈ $[t_8, t_9)$	0.2822	0.7487	0.1400001173	2243.38087	2243.22087	2244.08087	2244.08087	8.5835e-008	2244.08087	16171988
t∈ $[t_9, t_{10})$	0.2822	0.7487	0.140000203	2243.38087	2243.22087	2244.08087	2244.08087	1.4784e-007	2244.08087	16171988
t∈ $[t_{10}, t_{11})$	0.2822	0.7487	0.140000351	2243.38087	2243.22087	2244.08087	2244.08087	2.5464e-007	2244.08087	16171987
t∈ $[t_{11}, t_{12})$	0.2822	0.7487	0.140000606	2243.38087	2243.22087	2244.08087	2244.08087	4.386e-007	2244.08087	16171987
t∈ $[t_{12}, t_{13})$	0.2822	0.7487	0.140001044	2243.38087	2243.22087	2244.08088	2244.08087	7.5545e-007	2244.08087	16171986
t∈ $[t_{13}, t_{14})$	0.2822	0.7487	0.1400018	2243.38087	2243.22087	2244.08088	2244.08087	1.3012e-006	2244.08087	16171985
t∈ $[t_{14}, t_{15})$	0.2822	0.7487	0.1400031	2243.38087	2243.22087	2244.08089	2244.08087	2.2412e-006	2244.08087	16171982
t∈ $[t_{15}, t_{16})$	0.2822	0.7487	0.140005342	2243.38087	2243.22087	2244.0809	2244.08086	3.8603e-006	2244.08087	16171978
t∈ $[t_{16}, t_{17})$	0.2822	0.7487	0.1400092	2243.38087	2243.22087	2244.0809	2244.08086	6.6493e-006	2244.08087	16171970
t∈ $[t_{17}, t_{18})$	0.2822	0.7487	0.140015852	2243.38087	2243.22087	2244.0809	2244.08085	1.1453e-005	2244.08087	16171957
t∈ $[t_{18}, t_{19})$	0.2822	0.7487	0.140027305	2243.38087	2243.22087	2244.081	2244.08084	1.973e-005	2244.08087	16171934
t∈ $[t_{19}, t_{20})$	0.2822	0.7487	0.140047035	2243.38087	2243.22087	2244.0811	2244.08082	3.3988e-005	2244.08087	16171895
t=$t_{20} = T$	0.2822	0.7487	0.140081023	2243.38087	2243.22087	2244.08128	2244.08079	5.856e-005	2244.08089	16171828
Dynamic Subgame										
t	$\tau_1(t)$	$\tau_2(t)$	a(t)	$V_1(t)$	$V_2(t)$	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t∈ $[t_0, t_1)$	0.000324	0.91267	0.14	2243.436	2243.2093	2244.136	2244.0693	0.00803	2244.0786	16172000
t∈ $[t_1, t_2)$	0.08106	0.86216	0.148	2243.4186	2243.2128	2244.1587	2244.0647	0.01186	2244.07864	16172000
t∈ $[t_2, t_3)$	0.09327	0.86606	0.1599	2243.4161	2243.2125	2244.2155	2244.0526	0.02188	2244.07865	16172000
t∈ $[t_3, t_4)$	0.11564	0.87066	0.1818	2243.4116	2243.2122	2244.3204	2244.0304	0.04313	2244.0831	16172000
t∈ $[t_4, t_5)$	0.15894	0.87007	0.2249	2243.4031	2243.2122	2244.5276	2243.9873	0.09418	2244.1088	16172000
t∈ $[t_5, t_6)$	0.21931	0.8672	0.3191	2243.3919	2243.2124	2244.9873	2243.8933	0.23769	2244.2424	16172000
t∈ $[t_6, t_7)$	0.3064	0.79619	0.5568	2243.3768	2243.2174	2246.1607	2243.6607	0.61694	2245.0526	16172000
t∈ $[t_7, T]$	0.32804	0	1	2243.3732	·	2248.3732	·	0	2248.3732	16172000

Table 2.7: Percentage Changes Over Time of Related Variables under Static v.s Dynamic Taxes

–CD/Cassette Case ($a_{t_0} = 0.14$)

Static Subgame							
t	$V_1(t)(\%)$	$V_2(t)(\%)$	$U_1(t)(\%)$	$U_2(t)(\%)$	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	0	0	2.6737e-010	-4.9018e-011	7.9151e-007	7.5755e-012	-1.3583e-008
$t_1 - t_2$	0	0	4.2512e-010	-8.5113e-011	1.3633e-006	1.3814e-011	-2.3395e-008
$t_2 - t_3$	0	0	7.3259e-010	-1.4661e-010	2.3482e-006	2.3618e-011	-4.0298e-008
$t_3 - t_4$	0	0	1.2615e-009	-2.5222e-010	4.0445e-006	4.0106-011	-6.9410e-008
$t_4 - t_5$	0	0	2.1728e-009	-4.3492e-010	6.9663e-006	6.9516e-011	-1.1955e-007
$t_5 - t_6$	0	0	3.7432e-009	-7.4819e-010	1.1999e-005	1.1987e-010	-2.0592e-007
$t_6 - t_7$	0	0	6.4463e-009	-1.2896e-009	2.0667e-005	2.0632e-010	-3.5467e-007
$t_7 - t_8$	0	0	1.1103e-008	-2.2205e-009	3.5596e-005	3.5516e-010	-6.1088e-007
$t_8 - t_9$	0	0	1.913e-008	-3.8252e-009	6.1311e-005	6.1228e-010	-1.0522e-006
$t_9 - t_{10}$	0	0	3.294e-008	-6.5929e-009	1.056e-004	1.0539e-009	-1.8123e-006
$t_{10} - t_{11}$	0	0	5.6737e-008	-1.2032e-008	1.8189e-004	1.8154e-009	-3.1215e-006
$t_{11} - t_{12}$	0	0	9.7724e-008	-1.8855e-008	3.1329e-004	3.1269e-009	-5.3764e-006
$t_{12} - t_{13}$	0	0	1.6832e-007	-3.3664e-008	5.396e-004	5.3875e-009	-9.2605e-006
$t_{13} - t_{14}$	0	0	2.8992e-007	-5.8885e-008	9.2941e-004	9.2742e-009	-1.595e-005
$t_{14} - t_{15}$	0	0	4.9936e-007	-9.8972e-008	0.0016	1.5974e-008	-2.7473e-005
$t_{15} - t_{16}$	0	0	8.6011e-007	-1.72e-007	0.00276	2.7509e-008	-4.732e-005
$t_{16} - t_{17}$	0	0	1.4815e-006	-2.963e-007	0.00475e-004	4.7364e-008	-8.1508e-005
$t_{17} - t_{18}$	0	0	2.5519e-006	-5.1038e-007	0.00818	8.1529e-008	-1.404e-004
$t_{18} - t_{19}$	0	0	4.3959e-006	-8.7918e-007	0.01409	1.4028e-007	-2.4185e-004
$t_{19} - t_{20}(T)$	0	0	7.5729e-006	-1.519e-006	0.02427	2.4117e-007	-4.1664e-004
Dynamic Subgame							
t	$V_1(t)(\%)$	$V_2(t)(\%)$	$U_1(t)(\%)$	$U_2(t)(\%)$	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	-0.00077	0.00015	0.00101	-0.0002	5.71428	1.9607e-006	0
$t_1 - t_2$	-0.00011	-0.00001	0.00253	-0.00053	8.04054	1.426e-007	0
$t_2 - t_3$	-0.0002	-0.00001	0.00467	-0.00098	13.69606	1.9896e-004	0
$t_3 - t_4$	-0.00037	0	0.00923	-0.00192	23.70737	0.00115	0
$t_4 - t_5$	-0.00049	8.9158e-006	0.02048	-0.00418	41.88528	0.00595	0
$t_5 - t_6$	-0.00067	0.00022	0.05226	-0.01036	74.48135	0.0361	0
$t_6 - t_7$	-0.00016	.	0.0985	.	79.60737	0.1479	0
$t_7 - T$	0	.	0	.	0	0	0

Table 2.8: Movement over Time of Related Variables under Static vs. Dynamic Taxes

–CD/Cassette Case ($a_{t_0} = 0.25$)

Static Subgame										
t	τ_1	τ_2	a(t)	V_1	V_2	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t∈ $[t_0, t_1)$	0.30572	0.74833	0.25	2243.3769	2243.2209	2244.6269	2243.9709	0.123	2244.1349	16172000
t∈ $[t_1, t_2)$	0.30572	0.74833	0.373	2243.3769	2243.2209	2245.2419	2243.8479	0.326	2244.3679	16038583
t∈ $[t_2, t_3)$	0.30572	0.74834	0.699	2243.3769	2243.2209	2246.8720	2243.5219	0.7048	2245.8637	15684956
t∈ $[t_3, T)$	0.32804	1	1	2243.3732	·	2248.3732	·	0	2248.3732	16172000
Dynamic Subgame										
t	$\tau_1(t)$	$\tau_2(t)$	a(t)	$V_1(t)$	$V_2(t)$	$U_1(t)$	$U_2(t)$	$a'(t)$	W(t)	R(t)
t∈ $[t_0, t_1)$	0.24607	0.80026	0.25	2243.3871	2243.2171	2244.6371	2243.9671	0.1256	2244.1346	16172000
t∈ $[t_1, t_2)$	0.28703	0.79103	0.3756	2243.3801	2243.2178	2245.2582	2243.8422	0.3321	2244.3741	16172000
t∈ $[t_2, t_3)$	0.31833	0.78653	0.7077	2243.3748	2243.2181	2246.9134	2243.5104	0.7039	2245.9188	16172000
t∈ $[t_3, T)$	0.32804	0	1	2243.3732	·	2248.3732	·	0	2248.3732	16172000

Table 2.9: Percentage Changes of Related Variables Over Time under Static vs. Dynamic Taxes

–CD/Cassette Case ($a_{t_0} = 0.25$)

Static Subgame							
t	$V_1(t)$(%)	$V_2(t)$(%)	$U_1(t)$(%)	$U_2(t)$(%)	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	0	0	0.027399	-0.005481	49.2	0.010381	-0.824988
$t_1 - t_2$	0	0	0.072602	-0.014530	87.3995	0.066647	-2.204852
$t_2 - t_3$	-1.6943e-004	·	0.066814	·	43.061516	0.111741	3.105167
$t_3 - T$	0	·	0	·	0	0	0
Dynamic Subgame							
t	$V_1(t)$(%)	$V_2(t)$(%)	$U_1(t)$(%)	$U_2(t)$(%)	a(t)(%)	W(t)(%)	R(t)(%)
$t_0 - t_1$	-3.1592e-004	2.9547e-005	0.02767	-0.00557	50.24999	0.01067	0
$t_1 - t_2$	-2.3401e-004	1.4452e-005	0.07372	-0.014786	88.406	0.06883	0
$t_2 - t_3$	-7.1455e-005	·	0.06497	·	41.30281	0.10928	0
$t_3 - T$	0	·	0	·	0	0	0

Player 1 \ Player 2	Good 1	Good 2
Good 1	$V_1(t)+e_1$, $V_1(t)+e_1$	$V_1(t)$, $V_2(t)$
Good 2	$V_2(t)$, $V_1(t)$	$V_2(t)+e_2$, $V_2(t)+e_2$

Figure 2.1 : 2 by 2 Symmetrical Game

Figure 2.2: Phase Diagram for Evolutionary Game

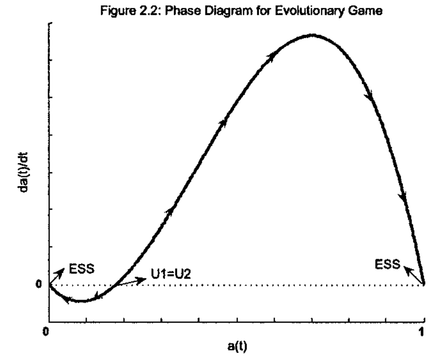


Figure 2.3: Optimal Dynamic Tax Rates for LCD TV and Tube TV (a_{t0}=0.192)

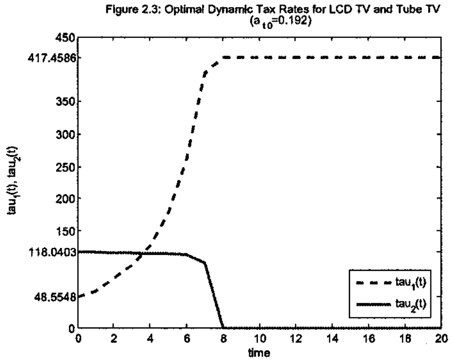


Figure 2.4: Static Tax Rates for LCD TV and Tube TV (a_{t0}=0.192)

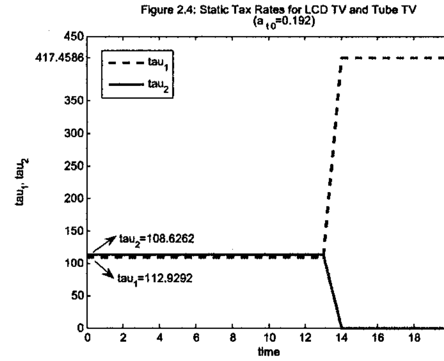


Figure 2.5: Evolution of Consumption of New Technology TV under Static vs. Dynamic Taxes (a_{t0}=0.192)

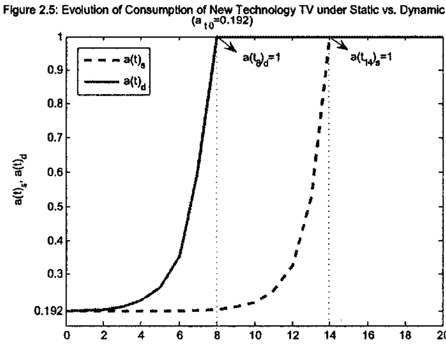


Figure 2.6: Movement of Social Welfare over Time under Static v.s Dynamic Taxes (a_{t0}=0.192)

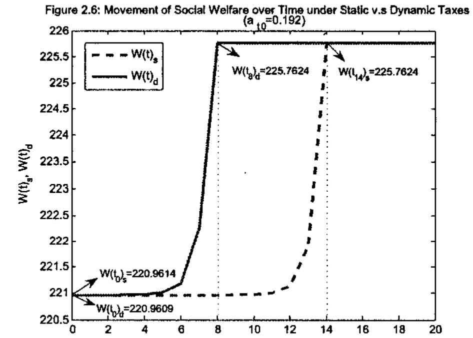


Figure 2.7: Optimal Dynamic Tax Rates for LCD TV and Tube TV $(a_{t0}=0.3)$

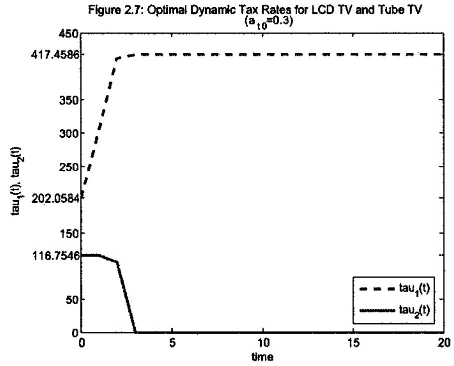


Figure 2.8: Static Tax Rates for LCD TV and Tube TV $(a_{t0}=0.3)$

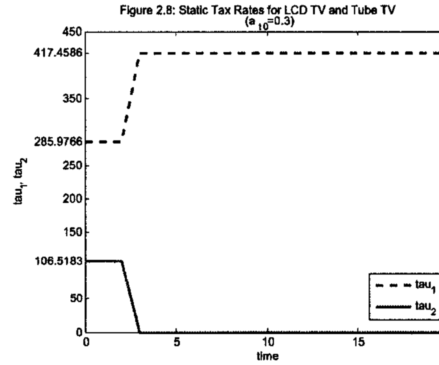


Figure 2.9: Evolution of Consumption of New Technology TV under Static vs. Dynamic Taxes $(a_{t0}=0.3)$

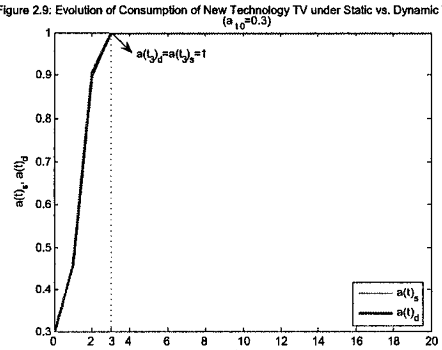
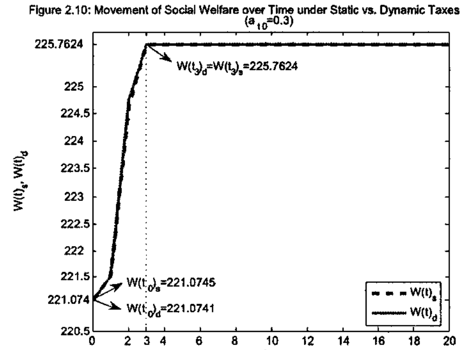


Figure 2.10: Movement of Social Welfare over Time under Static vs. Dynamic Taxes $(a_{t0}=0.3)$



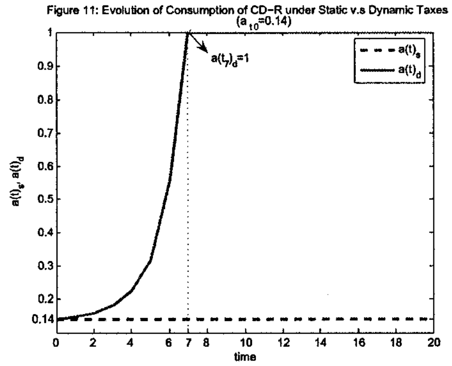


Figure 11: Evolution of Consumption of CD-R under Static v.s Dynamic Taxes (a_{t0}=0.14)

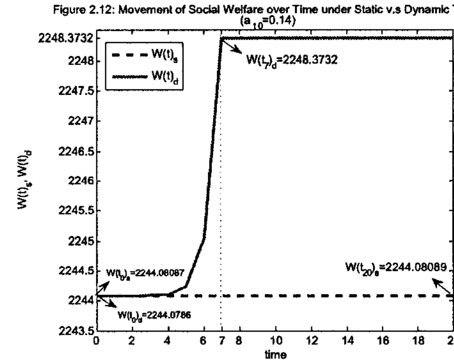


Figure 2.12: Movement of Social Welfare over Time under Static v.s Dynamic Taxes (a_{t0}=0.14)

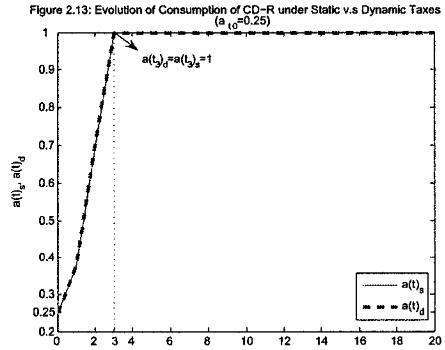


Figure 2.13: Evolution of Consumption of CD-R under Static v.s Dynamic Taxes (a_{t0}=0.25)

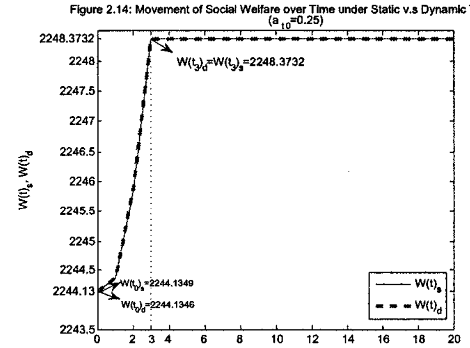


Figure 2.14: Movement of Social Welfare over Time under Static v.s Dynamic Taxes (a_{t0}=0.25)

3. GOVERNMENT FUNDING POLICY TOWARDS COMMUNICABLE DISEASES

3.1 INTRODUCTION

The paper analyzes the treatment towards communicable diseases under monopoly power in the presence of externalities and heterogenous agents. Our paper is motivated by the prevalence of communicable diseases such as malaria, tuberculosis and HIV/AIDS in developing countries, that often involves a monopoly firm in providing of pharmaceutical drug and government concern about the welfare of individuals in the population.

The economic literature on communicable diseases has been developing since the early of 1990's. Brito (1991) analyzes externalities associated with vaccination in a static framework with heterogenous agents and argues that it may not be correct for the government to compel all individuals to get vaccinated. Francis (1997) examines externalities in the market for vaccinations in a static and a dynamic environment respectively. Gersovitz and Hammer (2004) provide a general framework for the economics of infection and associated infection externality and prevention externality. The study reveals that in the SIS (susceptible-infected-susceptible) model, the optimal government subsidies should equally weight both preventive and therapeutic activities. However, these studies relate to market power-free framework. In contrast, Geoffard and Philipson (1997) first point out that a vaccine monopolist faces a non-standard dynamic incentive to keep the disease and thus to increase its profit. Their results show that a steady-state of infection may be compatible with a constant price. Recently, Mechoulan (2007) proposes a new theoretical framework for the dynamic problem of treatment under different market structures where externalities and heterogenous agents are present. The main results are the price and prevalence paths of a drug monopolist converge to a non-zero steady state, while the social planner generally eradicates the disease, or subsidizes treatments when eradication is impossible or too costly.

Infectious diseases are highly prevalent in developing countries. In sub-Saharan Africa, 400 million people suffer from malaria at any one time. Together, malaria, tuberculosis and HIV/AIDS kill six million people a year. By 2010, AIDS will have orphaned 20 million children, 18 million of them from sub-Saharan Africa (Swiss Agency for Development and Cooperation). The high mortality rate of these major communicable diseases in developing countries is associated with the fact that there is insufficient access to drugs and treatments in these countries. According to recent figures from the World Health Organization (WHO), 30 percent of the world's population lacks of access to life-saving medicines. In some countries in Asia and Africa, the number may be as high as 50 percent. To solve the problem, in recent years, the international community has attempted to encourage the local production of pharmaceutical drugs for some life-threatening communicable diseases to reduce transportation costs and to bolster research and development (R & D) as well. At the G8 2007 summit, the G8 recommitted its members to support "those African countries through technical assistance and capacity building programs to improve their access to affordable, safe, effective and high quality generic and innovative medicines" (Roger Bate, 2008). There is also support for local production among the board and staff of the Global Fund to Fight AIDS, Tuberculosis and Malaria, a public-private partnership that has allocated $10 billion to fight these dieases in poor and middle-income countries, mostly through the donations of western governments and financed by taxpayers (Roger Bate, 2008). In addition to public calls for aid-supported local production, some private companies are also moving investment in poor countries. For instance, the joint venture builds on a partnership between Uganda's Quality Chemicals and India's Cipla led to the construction of a new $38 million plant in

Kampala which is set to begin producing ARVs and antimalarials first time domestically in January 2008 (Roger Bate, 2008). Besides using the aid provided by the global funding, the governments in developing countries also offer tax incentives and subsidies to the pharmaceutical firms producing locally. As a result, the local prices for drugs and treatments have been reduced substantially in some countries. For example, the Thailand's HIV program becomes cheap by using the government-subsidized GPO-Vir (i.e., $24 per patient per month). Furthermore, the new treatments for malaria called "artemisinin-based combination therapies" (ACTs) currently cost about $2 per treatment course. It is recommended that $300 million to $500 million annual subsidy should make the price of ACTs in the range of 10 cents to 20 cents per course, which is more affordable for the impoverished people in many developing countries (Report from the Institute of Medicine of the National Academies).

This study extends the work of Stéphane Mechoulan (2007) by introducing the government and the monopoly's choice into the model and investigating the impact of government subsidy fund aimed at reducing production cost of the pharmaceutical drug on monopoly price for treatment, prevalence of diseases and thus on the total social welfare in a dynamic environment. The consumers have heterogeneous preferences over being healthy and the economic choice of the patients influences the spread of the disease. There is a foreign drug monopolist in the market producing treatment. The local government faces a choice of whether or not to offer a production subsidy to the monopolist to reduce the marginal cost of the drug.[11] If the government decides to offer the fund, the firm chooses to either

[11]The government would clearly offer an alternative subsidy, namely, a subsidy of the fixed cost of production. This latter scenario is explored in Appendix B.4. Here the subsidy of the unit cost is explored since the international aid may be aimed only at the production allocated to a specific developing country, so only a proportion of the firm's overall production. In this case, the incremental on marginal cost associated with production is materialized in the firm and funding agent's decisions. Nonetheless, both the subsidy of variable cost and fixed cost are formally considered in the thesis. For the latter, it is the fixed cost of a new entrant firm that is formally considered.

accept it or reject it. If the firm accepts the fund, the government subsidy is financed by taxes imposed on consumers.

The specific questions addressed by this study are as follows: First, under what conditions will the drug monopolist choose to accept the offer of the local government? How does the monopoly price for treatment behave when the number of sick in the population grows? Second, what parameters will influence the optimal level of government funding and what are the directions of change? Third, under what conditions does the local government offer subsidy for reducing production cost for treatment to the foreign drug monopoly in the market? These questions will be addressed through the development of an economic model, involving both the government's choice and the monopolist's choice in a dynamic environment given the prevalence path of the disease. The economy is represented by a sequential game involving: In stage 1, the government chooses whether or not to offer a production subsidy to the monopolist. If the government decides to do so, the optimal value of the government funding is determined at stage 2. In stage 3, the monopolist chooses whether or not to accept the offer provided by the government by comparing its profit with and without government subsidy. If the firm accepts the offer, the dynamic prices for treatment is determined by the monopolist given the ex-post cost of production starting at stage 4. In contrast, if the firm decides to reject the government funding, the market price for treatment is determined by the monopolist given the ex-ante cost of production from stage 4. Whereas if the government decides not to provide the fund in the first stage, the monopolist in the market producing at ex-ante cost determines the dynamic prices for treatment beginning from stage 2. The key findings of the study are as follows: First, the foreign drug monopolist takes the production subsidy of local government if its productivity type parameter is sufficiently high. Additionally, the monopoly price for treatment declines with the prevalence of the disease. Second, when the optimal value of government funding is greater than zero, the optimal level of the subsidy increases with the expected ex-ante cost of production and decreases with the expected type parameter of the firm. Further, the government would like to raise the level of fund with the growth in proportion of sick if the cost reduction associated with the government funding would sufficiently benefit consumers. Finally, the local government would be more likely to be involved by providing production subsidy to the foreign drug monopolist when the ex-ante cost of production and/or the type parameter of the firm are expected to be high and the number of sick in the population reaches a sufficiently large value. In empirical application, as compared with the numerical simulations used by Mechoulan (2007) to solve the dynamic problems, we use a newly developed toolbox of matlab for dynamic optimization, dynopt, to formally solve all dynamic optimization problems defined by the model. The empirical results show consistency with the predictions implied by the theoretical framework.

The paper proceeds as follows: The theoretical model is present in section 2 of the paper. In section 3, predictions implied by the theoretical model are examined in the empirical framework. The conclusion of the study is outlined in section 4.

3.2 MODEL

Consider an economy with communicable diseases. The consumers have heterogenous preferences over being healthy. The taste parameter for being healthy, β, is assumed to be uniformly distributed in the interval $[0, 1]$. The choice of patients is between paying for a treatment or not. The value of being sick and untreated is normalized to zero. Formally, as in Mechoulan (2007), let U denote the utility function of consumers at time period t:

$$U = \begin{cases} \beta & if \;\; healthy \\ \beta - p_t & if \;\; sick\ treated \\ 0 & if \;\; sick\ untreated \end{cases} \tag{3.1}$$

where $\beta \in [0, 1]$. Clearly, the individuals who are healthy do not buy the treatment and therefore receive higher expected utilities than those who are sick. Patients choose to be treated if and only if they are sick at time period t and have positive payoff by purchasing the treatment, denoted by: $\beta > p_t$. A simple transmission mechanism of the disease from one period to the next is proposed as in Mechoulan (2007): The disease strikes through person-to-person contacts(e.g., tuberculosis). The people may be reinfected immediately after recovery so that neither treatment nor natural recovery confers temporary immunity. Therefore, there is always a positive demand for treatment for any positive prevalence of the disease. For simplicity, it is assumed that a patient who has bought the treatment at time t can not transmit the disease to others at time $t+1$, so it is in that sense that the economic choice of the patients influences its spread. In other words, anyone may become sick in any period, regardless of the past, but prevalence[12] at time $t+1$ is still a function of the proportion of those sick who did not buy treatment in time period t. The size of the total population is normalized to one. Formally, let the proportion of sick at time t be r_t where $r_t \in [0, 1]$ and let $T(r_t)$ be an endogenous transmission function. Since the higher is the prevalence, the slower is the spread of the disease, $T(r_t)$ has the property of $T'(r_t) < 0$. As Mechoulan (2007), we define the indifferent patient by $\beta^* = p_t$. β^* represents the proportion of r_t sick patients who do not get treated at time period t. Consequently, the prevalence in the next period is defined as: $r_{t+1} = T(r_t)r_t p_t$. For simplicity, we assume that $T(r_t) = \alpha(1 - r_t)$, where: α is a positive parameter. The change rate of r_t over time can thus be approximated by $\lim\limits_{\Delta t \to 0} \dfrac{\Delta r_t}{\Delta t}$. Therefore, $r'(t) = r_t[\alpha(1 - r_t)p_t - 1]$ which is denoted by $g(r_t, p_t)$ in the sequel. It is obvious that $\dfrac{\partial g(r_t, p_t)}{\partial p_t} = \alpha[r_t(1 - r_t)] > 0$ for any $r_t \in (0, 1)$. Intuitively, high price for treatments increases the rate of infection. In addition, $\dfrac{\partial g(r_t, p_t)}{\partial r_t} = \alpha p_t(1 - 2r_t) - 1$ which is positive when $r_t < \dfrac{\alpha p_t - 1}{2\alpha p_t}$.

[12]Prevalence is defined as percentage of sick in the population.

It is assumed that there is a foreign monopolist in the local market producing treatment with marginal cost c_1, where $0 < c_1 < 1$, and potentially producing for other regional markets. The production cost of the firm c_1 is not known by the government, but it is assumed to be uniformly distributed in the interval $[\underline{c}, \overline{c}]$. The local government faces a choice of whether or not to offer a subsidy to reduce the production cost of the foreign firm. The ex-post cost of the monopolist after receiving the government funding depends on the value of the firm's productivity type parameter θ and the level of government subsidy w where θ is distributed uniformly on the interval $[\underline{\theta}, \overline{\theta}]$. The firm could either accept or reject the offer of the government.[13] If the firm accepted the offer, the value of government funding would be compensated by collecting a constant tax rate τ per period from consumers. As a social planner, the government is concerned about the social welfare which is assumed to be of the Benthamite social welfare form. The social welfare at time period t is represented by: $W_t(U_1, U_2) = r_t V_1(p_t) + (1 - r_t)V_2$ where $V_1(p_t)$ denotes the expected indirect utility of patients who get treated at time period t and V_2 denotes the expected utility of individuals who are healthy at time t. The utility of those who are sick but do not purchase the treatment at t is normalized to zero. Clearly, $V_1(p_t) < V_2$. The economy is represented by a sequential game in a dynamic environment. The sequence of the game is as follows: In stage 1, the government chooses whether or not to offer a production subsidy to the monopolist. If the government decides to do so, the optimal value of the subsidy fund is determined at stage 2. In stage 3, the monopolist chooses whether or not to accept the offer provided by the government by comparing its profit with and without government funding. If the firm accepts the offer, the dynamic prices for treatment is determined by the monopolist given the ex-post cost of production $c(w, \theta)$ starting at stage 4. In contrast, if the firm decides to refuse the

[13]The government subsidy can be considered to be a direct payment to the firm or the government providing a grant to the firm to cover a specific component of the cost associated with the treatment. For instance, malaria, tuberculosis and HIV/AIDS treatments involve a pharmaceutical drug, a treatment regime and drug delivery mechanism. These are all costs associated with treatments and, therefore, constitute the parameter c_1.

government funding, the market price for treatment is determined by the monopolist given the ex-ante cost of production c_1 from stage 4. Whereas if the government decides not to provide the production subsidy in the first stage, the monopolist in the market producing at cost c_1 determines the dynamic prices for treatment beginning from stage 2.

The sequential game is solved through backward induction. For the subgame with government subsidy, staring at stage 4, the monopolist determines the optimal dynamic prices for treatment subject to the dynamics over time of the prevalence of the disease given the ex-post cost of production $c(w, \theta)$ if the firm accepts the offer and the ex-ante cost c_1 if the firm rejects the offer respectively. In stage 3, the monopolist decides whether or not to accept the government funding by comparing the profits with and without the fund w. The optimal value of the production subsidy is determined by the government at stage 2 by maximizing the expected value of aggregate social welfare subject to the participation constraint of the firm given the prevalence path of the disease. While for the subgame in which the government decides not to provide the fund, the monopolist in the market chooses the optimal prices for treatment dynamically starting from stage 2 given the marginal cost of production c_1. In what follows, the subgame with government involvement is first solved and the subgame without government intervention is next. The government choice of whether or not to offer a subsidy to the foreign drug monopolist for cost deduction takes place in stage 1 of the game.

3.2.1 SUBGAME WITH GOVERNMENT INTERVENTION

If the government decided to provide the subsidy funding for cost deduction to the foreign drug monopolist, the firm could either accept it or reject it. Let $j \in (a, b)$ denote the action space for the firm, where: a represents accept and b represents reject. In the subgame with government funding, if the firm takes the offer (i.e., $j = a$), the corresponding ex-post cost of production is defined as: $c(w,\theta) = c_1 - \theta w$, where c_1 is uniformly distributed on $[\underline{c}, \overline{c}]$ with $0 < \underline{c} < \overline{c} < 1$ and θ is uniformly distributed on $[\underline{\theta}, \overline{\theta}]$ with $0 < \underline{\theta} < \overline{\theta} < 1$. Clearly, $c(w,\theta) < c_1$ for any values of $w > 0$. Therefore,, the ex-post cost of production is negatively correlated with the type parameter of the firm θ and the government funding w (i.e., $\frac{dc(w,\theta)}{d\theta} < 0$ and $\frac{dc(w,\theta)}{dw} < 0$). Given w, the higher the θ, the lower is the $c(w,\theta)$. Similarly, given θ, the more government subsidy reduces the ex-post cost of production more. While if the firm rejects the government funding (i.e., $j = b$), the production cost of the firm is unchanged, equal to the ex-ante cost c_1. Starting at stage 4, the monopolist maximizes the value of aggregate profit by choosing the optimal dynamic prices for treatment given the prevalence path of the disease.

3.2.1.1 SUBGAME INVOLVING THE FIRM ACCEPTING THE PRODUCTION SUBSIDY OF GOVERNMENT

If the firm accepts the offer of government, a constant tax rate τ will be collected from every consumer per time period to compensate the value of production subsidy of government. The utility function of consumers at time period t becomes:

$$U = \begin{cases} \beta - \tau & if \;\; healthy \\ \beta - p_t - \tau & if \;\; sick\ treated \\ 0 - \tau & if \;\; sick\ untreated \end{cases} \tag{3.2}$$

The demand function for the treatment at time period t is derived as: $D(p_t + \tau) = r_t \int_{p_t+\tau}^{1} d\beta = r_t(1 - p_t - \tau)$. And the dynamics over time of the prevalence of disease is characterized by the function $r'(t) = r_t[\alpha(1 - r_t)(p_t + \tau) - 1]$. Let we assume that after time period T, the competitive price applies, so there is no profit to be made after time T. We solve the problem through backward induction.

Starting at stage 4, given the ex-post cost of production $c(w, \theta)$, the monopolist chooses the optimal dynamic prices for treatment to maximize the value of aggregate profit. The firm' s problem is as follows:

$$(3.3) \quad \max_{\{p_t\}} \int_0^T e^{-rt} \{r_t[p_t - c(w,\theta)](1 - p_t - \tau)\} \, dt$$
$$s.t. \ (1) \ r_t' = r_t[\alpha(1 - r_t)(p_t + \tau) - 1]$$

As shown in Appendix B.1, the optimal dynamic prices for treatment by taking the government production subsidy are characterized as follows:

$$(3.4) \quad p_t = \frac{1}{2}[1 + c(w,\theta) - \tau + \mu_2(t)\alpha(1 - r_t)]$$

where: $\mu_2(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate profit of the firm (i.e., the shadow price of r_t). As shown in Appendix B.1, $\mu_2(t) > 0$ so the increase in the number of sick in the population benefits the monopolist. Since the total tax revenue should compensate the value of government funding, it follows that $\int_0^T e^{-rt}\tau = w$. The level of constant tax rate τ is derived as: $\tau = \frac{rw}{1 - e^{-rT}}$, where r is the discount factor and $1 - e^{-rT}$ is positive and close to 1 by assuming r is sufficiently small and T is large enough. Given $c(w, \theta) = c_1 - \theta w$ and $\tau = \frac{rw}{1 - e^{-rT}}$, the dynamic prices for treatment can thus be rewritten as:

$$(3.5) \quad p_t = \frac{1}{2}[1 + c_1 - (\theta + \frac{r}{1 - e^{-rT}})w + \mu_2(t)\alpha(1 - r_t)]$$

Given equation 3.5, the profit function of the monopoly at time period t can thus be obtained as follows:

$$(3.6) \qquad \pi_t = \frac{r_t}{4}\left[\left(1 - c_1 + (\theta - \frac{r}{1-e^{-rT}})w\right)^2 - \alpha^2(1-r_t)^2(\mu_2(t))^2\right]$$

Let $p_t(r_t, c_1, w, \theta; a)$ and $\pi_t(r_t, c_1, w, \theta; a)$ represent the optimal dynamic prices for treatment and the payoff functions of the monopolist at time t respectively for the subgame in which the firm accepts the offer of government in the sequel.

3.2.1.2 SUBGAME INVOLVING THE FIRM REJECTING THE PRODUCTION SUBSIDY OF GOVERNMENT

If the firm decides to reject the offer of the government, the production cost of the firm is unchanged, equal to the original marginal cost of production c_1. The market demand for treatment is derived as $D(p_t) = r_t \int_{p_t}^{1} d\beta = r_t(1-p_t)$. [14] Additionally, the equation characterizing the dynamics over time of the proportion of sick is given by $r_t' = r_t[\alpha(1-r_t)p_t - 1]$. Starting at stage 4, the monopolist maximizes the present value of aggregate profit by choosing the optimal dynamic prices for treatment given the prevalence path of the disease as follows:

$$(3.7) \qquad \max_{\{p_t\}} \int_0^T e^{-rt}\left\{r_t(p_t - c_1)(1-p_t)\right\} dt$$
$$s.t. \ (1) \ r_t' = r_t[\alpha(1-r_t)p_t - 1]$$

As shown in Appendix B.2, the optimal dynamic prices for treatment by rejecting the government production subsidy is characterized as follows:

$$(3.8) \qquad p_t = \frac{1}{2}[1 + c_1 + \mu_3(t)\alpha(1-r_t)]$$

[14] As a reminder that the utility function for those sick and get treated at time t is $\beta - p_t$ where $\beta \sim U[0,1]$.

where: $\mu_3(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate profit. Similar to $\mu_2(t)$, as shown in Appendix B.2, $\mu_3(t)$ is also positive. It follows that the aggregate profit of the monopolist raises with the growth in proportion of sick. Given equation 3.8, the profit function of the firm at time t can thus be derived as follows:

$$\pi_t = \frac{r_t}{4}\left[(1-c_1)^2 - \alpha^2(1-r_t)^2(\mu_3(t))^2\right] \tag{3.9}$$

In the sequel, let $p_t(r_t, c_1, w, \theta; b)$ and $\pi_t(r_t, c_1, w, \theta; b)$ denote the optimal dynamic prices for treatment and the payoff functions of the monopolist at time t respectively for the subgame in which the firm rejects the offer of the government. [15]

Given the government funding w, in stage 3, the firm decides whether or not to take the offer of the government. The decision depends on the result of comparison of the aggregate profits with and without production subsidy. As shown above, $\pi_t(r_t, c_1, w, \theta; a)$ and $\pi_t(r_t, c_1, w, \theta; b)$ represent the optimal profits of the firm at time t by taking and not taking the subsidy fund respectively. Clearly, the firm accepts the government's offer if and only if $\int_0^T e^{-rt}[\pi_t(r_t, c_1, w, \theta; a)]dt \geq \int_0^T e^{-rt}[\pi_t(r_t, c_1, w, \theta; b)]dt$. By comparing $\pi_t(r_t, c_1, w, \theta; a) = r_t[1-p_t(r_t, c_1, w, \theta; a)-\tau][p_t(r_t, c_1, w, \theta; a)-c(w, \theta)]$ and $\pi_t(r_t, c_1, w, \theta; b) = r_t[1-p_t(r_t, c_1, w, \theta; b)][p_t(r_t, c_1, w, \theta; b)-c_1]$, it is apparently that by accepting the government funding, the production cost of the firm reduces (i.e., $c(w, \theta) < c_1$ for any positive w and θ). However, the demand for treatment also declines if the firm accepts the offer due to the added tax τ imposed by the government. As a result, $\pi_t(r_t, c_1, w, \theta; a) > \pi_t(r_t, c_1, w, \theta; b)$

[15] Note that the values of the government funding w and the type parameter of the firm θ do not influence the dynamic prices and the profits of the monopolist when the firm does not take the production subsidy (i.e., $\frac{dp_t(r_t,c_1,w,\theta;b)}{dw} = 0$, $\frac{dp_t(r_t,c_1,w,\theta;b)}{d\theta} = 0$, $\frac{d\pi_t(r_t,c_1,w,\theta;b)}{dw} = 0$ and $\frac{d\pi_t(r_t,c_1,w,\theta;b)}{d\theta} = 0$).

if the positive impact of the decrease in production cost dominates the negative impact of the reduce in demand associated with the government production subsidy on the profit of the firm and vice versa. Further, $\pi_t(r_t, c_1, w, \theta; a) = \pi_t(r_t, c_1, w, \theta; b)$ if the growth in profit due to the cost deduction just equals to the decline in profit due to the decrease in demand. Given equation 3.6 and equation 3.9, clearly, $\pi_t(r_t, c_1, w, \theta; a) = \pi_t(r_t, c_1, w, \theta; b)$ when $\theta = \frac{r}{1-e^{-rT}}$. As shown in Appendix B.1, $\pi_t(r_t, c_1, w, \theta; a)$ grows with θ. It is clear that $\pi_t(r_t, c_1, w, \theta; a) < \pi_t(r_t, c_1, w, \theta; b)$ for $\theta < \frac{r}{1-e^{-rT}}$, and $\pi_t(r_t, c_1, w, \theta; a) \geq \pi_t(r_t, c_1, w, \theta; b)$ for $\theta \geq \frac{r}{1-e^{-rT}}$. Consequently, at stage 3, the firm accepts the production subsidy of government if its productivity type parameter θ is not smaller than $\frac{r}{1-e^{-rT}}$ and rejects the offer otherwise.

The optimal value of government funding is determined at stage 2. The utility function of consumers is represented by equation 3.2 if the firm takes the offer of government subsidy. As a result, the expected indirect utility of patients who get treated at time t is first derived as $V_1(p_t+\tau) = \int_{p_t+\tau}^{1} (\beta - p_t - \tau) f(\beta) d\beta = \frac{1}{2} + \frac{1}{2}(p_t+\tau)^2 - (p_t+\tau)$. Second, the expected indirect utility of those who are sick but do not purchase treatment at time t is derived as $V_1(\tau) = \int_{0}^{p_t+\tau} (0-\tau) f(\beta) d\beta = -\tau(p_t+\tau)$. Last, the expected indirect utility of consumers who are healthy at t is $V_2(\tau) = \int_0^1 (\beta - \tau) f(\beta) d\beta = \frac{1}{2} - \tau$. The consumers who are healthy always have a higher expected payoff than the sick people (i.e., $V_2(\tau) > V_1(p_t+\tau)$), thus, the consumer price for treatment needs to satisfy the following assumption: $p_t + \tau < \min\{1, \sqrt{2p_t}\}$. As the ex-ante cost of the firm c_1 and the type parameter θ are not known by the government, the government chooses the optimal value of production subsidy to maximize the expected value of total social welfare. The prevalence path of disease is characterized by the equation $r_t' = r_t\left[\alpha(1-r_t)(p_t+\tau) - 1\right]$. Given $p_t = p_t(r_t, c_1, w, \theta; a)$, the function of optimal dynamic prices for the monopolist by accepting the subsidy fund, the government's problem can be summarized as follows:

$$\max_{\{w\}} E_{c_1}E_\theta\left\{\int_0^T e^{-rt}\left[r_t(V_1(p_t(r_t,c_1,w,\theta;a)+\tau)+V_1(\tau))+(1-r_t)V_2(\tau)\right]dt\right\} \tag{3.10}$$
$$s.t.\ (1)\ r_t' = r_t\left[\alpha(1-r_t)(p_t(r_t,c_1,w,\theta;a)+\tau)-1\right]$$
$$(2)\ \pi_t(r_t,c_1,w,\theta;a) \geq \pi_t(r_t,c_1,w,\theta;b)$$

In which (2) is the constraint that the firm takes the offer provided by the government. The firm's participation constraint is binding only at $\theta = \frac{r}{1-e^{-rT}}$.

As shown in Appendix B.1, the optimal value of government production subsidy is as follows:

$$w = \frac{\left(\frac{\overline{\theta}+\underline{\theta}}{8} - \frac{r}{4(1-e^{-rT})}\right)A_1 - \frac{\overline{\theta}+\underline{\theta}}{4} + \frac{r}{r_t(1-e^{-rT})} + A_2}{\frac{1}{12}(\overline{\theta}^2+\overline{\theta}\underline{\theta}+\underline{\theta}^2) + \frac{r(\underline{\theta}+\overline{\theta})}{4(1-e^{-rT})} - \frac{3r^2}{4(1-e^{-rT})^2} - \frac{\lambda_1(t)}{2}\left(\theta - \frac{r}{(1-e^{-rT})}\right)^2} \tag{3.11}$$

where: $A_1 = \left[1+\frac{\overline{c}+\underline{c}}{2}+\alpha(1-r_t)(\mu_2(t)+2\mu_1(t))\right]$ and $A_2 = \frac{r\left(\frac{\underline{c}+\overline{c}}{2}+\alpha\mu_2(t)(1-r_t)\right)}{2(1-e^{-rT})} + \frac{\lambda_1(t)}{2}(1-c_1)\left(\theta-\frac{r}{(1-e^{-rT})}\right)$. $\mu_1(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the expected value of aggregate social welfare. As shown in Appendix B.1, $\mu_1(t) < 0$, the aggregate utility of consumers is expected to decline with prevalence of the disease. Therefore, the consumers are worse off when the number of sick in the population increases. [16] The firm accepts the offer if and only if $\theta \geq \frac{r}{1-e^{-rT}}$. It follows that $\lambda_1(t) > 0$ for $\theta = \frac{r}{1-e^{-rT}}$ and $\lambda_1(t) = 0$ for $\theta > \frac{r}{1-e^{-rT}}$. The function of optimal subsidy fund of government for $\theta \geq \frac{r}{1-e^{-rT}}$ can thus be rewritten as:

[16] As a reminder, $\mu_2(t)$ is the parameter measuring the impact of increase in r_t on the aggregate profit which is positive. $\lambda_1(t)$ is the Lagrangian multiplier associated with the participation constraint of the firm.

$$(3.12) \qquad w = \frac{\left(\frac{\overline{\theta}+\underline{\theta}}{8} - \frac{r}{4(1-e^{-rT})}\right) A_1 - \frac{\overline{\theta}+\underline{\theta}}{4} + \frac{r}{r_t(1-e^{-rT})} + B_2}{\frac{1}{12}(\overline{\theta}^2 + \overline{\theta}\underline{\theta} + \underline{\theta}^2) + \frac{r(\underline{\theta}+\overline{\theta})}{4(1-e^{-rT})} - \frac{3r^2}{4(1-e^{-rT})^2}}$$

where: $B_2 = \frac{r}{2(1-e^{-rT})}\left[\frac{\underline{c}+\overline{c}}{2} + \alpha\mu_2(t)(1-r_t)\right]$. The government production subsidy is taken only when $\theta \geq \frac{r}{1-e^{-rT}}$, hence, $w > 0$ for $\theta \geq \frac{r}{1-e^{-rT}}$ and $w = 0$ for $\theta < \frac{r}{1-e^{-rT}}$. Given the discount factor r is sufficiently small and the termination time period T is long enough, the denominator of equation 3.12 is positive. To make the numerator of equation 3.12 be greater than zero, the value of $1 + \frac{\overline{c}+\underline{c}}{2} + \alpha(1-r_t)(\mu_2(t) + 2\mu_1(t))$ needs to be more positive when r_t is large and can be less positive when r_t is small. Since $0 < \underline{c} < \overline{c} < 1$ and $\alpha > 0$, it implies that $\mu_2(t) + 2\mu_1(t) < 0$.

Given the optimal value of the production subsidy w for $\theta \geq \frac{r}{1-e^{-rT}}$, the optimal dynamic prices for treatment for the subgame in which the firm accepts the government fund, $p_t(r_t, c_1, w, \theta; a)$, is obtained first through equation 3.5. Next, the indirect utility of consumers at time t can be calculated by substituting $p_t(r_t, c_1, w, \theta; a)$ and τ into the utility functions we derived above. As c_1 and θ are only known by the firm, the expected payoff for the consumers are thus represented as $E_{c_1}E_\theta[V_1(p_t(r_t, c_1, w, \theta; a) + \tau)]$ for patients who get treated at time t, $E_{c_1}E_\theta[V_1(\tau)]$ for patients who are sick but do not buy treatment at time t and $V_2(\tau)$ for those who are healthy at t. Using the expected indirect utility of consumers, the expected aggregate payoff of government for the subgame the monopolist takes the production subsidy of government is finally obtained which is denoted as $E_{c_1}E_\theta\left\{\int_0^T e^{-rt}\left[r_t(V_1(p_t(r_t, c_1, w, \theta; a) + \tau) + V_1(\tau)) + (1-r_t)V_2(\tau)\right] dt\right\}$. In contrast, for $\theta < \frac{r}{1-e^{-rT}} = \hat{\theta}$,[17] the firm rejects the offer of government, therefore $w = 0$. The market

[17]The lower productivity type firm has no incentive to mimic the higher productivity types. First, for the firm with a productivity type sufficiently small ($\theta < \hat{\theta}$), the firm can obtain higher level of profit by rejecting the offer of government as opposed to accepting it, as implied by the participation constraint of the firm in Equation 3.10. Second, for the firm with a productivity

price for treatment is determined by the monopolist at the ex-ante cost c_1 and the function of $p_t(r_t, c_1, w, \theta; b)$ is given by equation 3.8. Correspondingly, the expected utility of consumers and thus the expected total social welfare can be calculated given $p_t(r_t, c_1, w, \theta; b)$. Let $E_{c_1}[V_1(p_t(r_t, c_1, w, \theta; b))]$ denote the expected payoff for patients who get treated at time t, V_2 denote the expected payoff for consumers who are healthy at time t (Remind that the expected payoff for patients who do not purchase treatment at t is normalized to zero.) and $E_{c_1}\left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1, w, \theta; b)) + (1-r_t)V_2\right]dt\right\}$ denote the total expected payoff of government for the subgame in which the firm rejects the production subsidy of the government.

From above, the equilibrium outcomes and the equilibrium payoffs for the agents for the subgame with government intervention are summarized as follows:

Given $j = a$ if $\theta \geq \dfrac{r}{1-e^{-rT}}$ and $j = b$ if $\theta < \dfrac{r}{1-e^{-rT}}$. First, the optimal dynamic prices for treatment at time t are:

$$(3.13) \qquad p_t(r_t, c_1, w, \theta; j) = \begin{cases} \frac{1}{2}[1 + c_1 - (\theta + \frac{r}{1-e^{-rT}})w + \mu_2(t)\alpha(1-r_t)] & if \quad j = a \\ \frac{1}{2}[1 + c_1 + \mu_3(t)\alpha(1-r_t)] & if \quad j = b \end{cases}$$

Second, the optimal aggregate profits for the monopoly are:

$$(3.14) \int_0^T e^{-rt}[\pi_t(r_t, c_1, w, \theta; j)]dt = \begin{cases} \int_0^T e^{-rt}\{r_t[p_t(r_t, c_1, w, \theta; a) - c(w, \theta)](1 - p_t(r_t, c_1, w, \theta; a) - \tau)\}dt & if \quad j = a \\ \int_0^T e^{-rt}\{r_t[p_t(r_t, c_1, w, \theta; b) - c_1](1 - p_t(r_t, c_1, w, \theta; b))\}dt & if \quad j = b \end{cases}$$

Third, the expected utilities for consumers at time t are:

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type higher than $\hat{\theta}$, the firm also does not have incentive to mimic other firms with higher types. For instance, consider firm 1 and firm 2 having productivity types θ_1 and θ_2 respectively, where: $\theta_2 > \theta_1 > \hat{\theta}$. Firm 1 would receive a lower level of government subsidy if it misreported its type to be higher than its true value since production subsidy w is a decreasing function of type parameter θ (i.e., $\frac{dw}{d\theta} < 0$). As a result, firm 1's ex-post cost of production, $c(w, \theta)$, would be reduced less by reporting its type to be θ_2 given $c(w, \theta) = c_1 - \theta w$ (i.e., $c_1 - \theta_1 w(\theta_2) > c_1 - \theta_1 w(\theta_1)$ where $w(\theta_2) < w(\theta_1)$). It follows that firm 1 earns a lower level of profit if it reports its type to be higher than its true type. Consequently, lower productivity types do not have an incentive to copy the strategy of higher productivity type firms.

For patients who get treated at time t: [18]

$$(3.15) E[U_{1t}(r_t, c_1, w, \theta; j)] = \begin{cases} E_{c_1} E_\theta[\frac{1}{2} + \frac{1}{2}(p_t(r_t, c_1, w, \theta; a) + \tau)^2 - (p_t(r_t, c_1, w, \theta; a) + \tau)] & if \quad j = a \\ E_{c_1}[\frac{1}{2} + \frac{1}{2}(p_t(r_t, c_1, w, \theta; b))^2 - p_t(r_t, c_1, w, \theta; b)] & if \quad j = b \end{cases}$$

For patients who do not buy treatment at time t:

$$(3.16) \qquad E[U_{2t}(r_t, c_1, w, \theta; j)] = \begin{cases} E_{c_1} E_\theta[V_1(\tau)] = E_{c_1} E_\theta\{-\tau[p_t(r_t, c_1, w, \theta; a) + \tau]\} \; if \; j = a \\ 0 \quad if \quad j = b \end{cases}$$

Next, for consumers who are healthy at time t:

$$(3.17) \qquad E[U_{3t}(r_t, c_1, w, \theta; j)] = \begin{cases} V_2(\tau) = \frac{1}{2} - \tau \quad if \quad j = a \\ V_2 = \frac{1}{2} \quad if \quad j = b \end{cases}$$

Last, the expected aggregate social welfare for the government are:

$$(3.18) E\int_0^T e^{-rt}[W_t(r_t, c_1, w, \theta; j)]dt = \begin{cases} E_{c_1} E_\theta \left\{\int_0^T e^{-rt}\left[r_t(V_1(p_t(r_t, c_1, w, \theta; a) + \tau) + V_1(\tau)) + (1 - r_t)V_2(\tau)\right] dt\right\} if \; j = a \\ E_{c_1} \left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1, w, \theta; b)) + (1 - r_t)V_2\right] dt\right\} \quad if \quad j = b \end{cases}$$

3.2.2 SUBGAME WITHOUT GOVERNMENT INTERVENTION

If the government decides not to involve in the first stage, the foreign monopolist in the local market determines the dynamic prices for treatment starting from stage 2. The firm produces at marginal cost c_1. The demand for treatment is $D(p_t) = r_t(1 - p_t)$. The problem of the firm involving the choice of the optimal dynamic prices is as follows: [19]

[18] To be consistent with the notations within the paper, $E[U_{1t}(r_t, c_1, w, \theta; j)] = E_{c_1} E_\theta[V_1(p_t(r_t, c_1, w, \theta; a) + \tau)]$ if $j = a$ and $E[U_{1t}(r_t, c_1, w, \theta; j)] = E_{c_1}[V_1(p_t(r_t, c_1, w, \theta; b))]$ if $j = b$.

[19] This problem is similar to the monopoly problem solved in Mechoulan (2007).

$$(3.19) \qquad \pi_{c_1} \equiv \max_{\{p_t\}} \int_0^T e^{-rt} \left\{ r_t(p_t - c_1)(1 - p_t) \right\} dt$$
$$s.t. \ (1) \ r_t' = r_t[\alpha(1 - r_t)p_t - 1]$$

As shown in Appendix B.2, the optimal dynamic prices for treatment without government intervention is characterized as follows:

$$(3.20) \qquad p_t = \frac{1}{2}[1 + c_1 + \mu_3(t)\alpha(1 - r_t)]$$

where: $\mu_3(t) > 0$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate profit. Apparently, for the subgame without government intervention, the firm's problem is as same as that for the subgame in which the government involves but the firm rejects the offer of the government subsidy. Let $p_t(r_t, c_1)$ denote the optimal dynamic prices for treatment without government intervention. Clearly, $p_t(r_t, c_1) = p_t(r_t, c_1, w, \theta; b)$. Given $p_t(r_t, c_1)$, the aggregate profit of the firm is derived and is represented as: $\int_0^T e^{-rt}[\pi_t(r_t, c_1)]dt$. The utility function of consumers is characterized by equation 3.1. Let $E_{c_1}[V_1(p_t(r_t, c_1))]$ denote the expected utility for patients who get treated at time t, V_2 denote the expected utility for consumers who are healthy at time t and the expected utility for patients who do not purchase treatment at t is normalized to zero. The expected aggregate social welfare for the subgame without government intervention is then calculated and is represented as: $E_{c_1}\left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1)) + (1 - r_t)V_2\right] dt\right\}$. Given $p_t(r_t, c_1) = p_t(r_t, c_1, w, \theta; b)$, clearly, the payoffs for the agents for the subgame in which the government does not involve are equivalent with those for the subgame in which the firm rejects the offer of the government. [20]

At stage 1, the government decides whether or not to offer the subsidy to the foreign drug monopolist to reduce the production cost of the drug. Clearly, the decision depends on the result of the comparison of the expected values of the aggregate social welfare with and without government intervention. The monopolist accepts the offer of government funding if and only if $\theta \geq \dfrac{r}{1-e^{-rT}}$. It follows that the expected value of aggregate social welfare when the firm rejects the offer is equivalent to that when the government does not involve: $E_{c_1}\left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1, w, \theta; b)) + (1-r_t)V_2\right]dt\right\} = E_{c_1}\left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1)) + (1-r_t)V_2\right]dt\right\}$. Therefore, when the productivity type of the firm is expected to be sufficiently low (i.e., $E(\theta) < \dfrac{r}{1-e^{-rT}}$), the government will not involve. Whereas when $E(\theta) \geq \dfrac{r}{1-e^{-rT}}$, the government chooses to provide a production subsidy for cost deduction to the foreign drug monopolist if and only if the following holds:

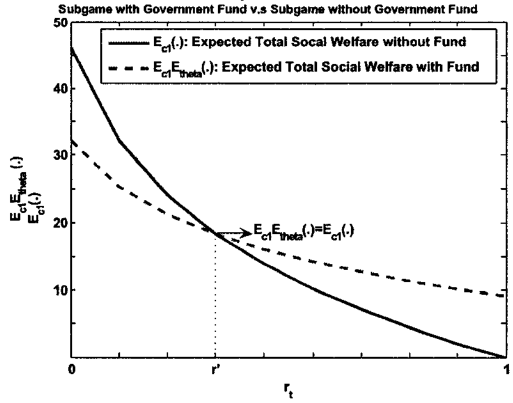
$$(3.21)\qquad E_{c_1}E_\theta\left\{\int_0^T e^{-rt}\left[r_t(V_1(p_t(r_t, c_1, w, \theta; a)+\tau)+V_1(\tau))+(1-r_t)V_2(\tau)\right]dt\right\} \geq E_{c_1}\left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1)) + (1-r_t)V_2\right]dt\right\}$$

where the dynamics over time of the proportion of sick is characterized by the function $r'_t = r_t\left[\alpha(1-r_t)(p_t(r_t, c_1, w, \theta; a)+\tau)-1\right]$ for the subgame with government subsidy, while $r'_t = r_t\left[\alpha(1-r_t)p_t(r_t, c_1)-1\right]$ represents the prevalence path of the disease for the subgame in which the government does not involve. If the production cost of the firm could be reduced sufficiently by taking the government subsidy such that $p_t(r_t, c_1, w, \theta; a)+\tau < p_t(r_t, c_1)$, then,

[20] That is: $\int_0^T e^{-rt}[\pi_t(r_t, c_1)]dt = \int_0^T e^{-rt}[\pi_t(r_t, c_1, w, \theta; b)]dt$, $E_{c_1}[V_1(p_t(r_t, c_1))] = E_{c_1}[V_1(p_t(r_t, c_1, w, \theta; b))]$ and $E_{c_1}\left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1)) + (1-r_t)V_2\right]dt\right\} = E_{c_1}\left\{\int_0^T e^{-rt}\left[r_t V_1(p_t(r_t, c_1, w, \theta; b)) + (1-r_t)V_2\right]dt\right\}$.

clearly, the patients could be better off due to the lower prices for treatment under government intervention. Moreover, the rate of infection would also be reduced with the lower prices for treatment since $\frac{dr'_t}{dp_t} > 0$. However, the government needs to collect taxes from consumers to raise the fund for subsidy which also induces the utility loss for consumers. The disutility of consumers by paying for the taxes is high when r_t is small since the proportion of consumers in the population who will benefit from the decline in price for treatments associated with the government funding is small. While with the growth in r_t, the number of consumers who have to purchase the treatment increases. More and more consumers will benefit from the government involvement. Consequently, the government may consider to involve by offering a subsidy fund to reduce the production cost of the drug when the proportion of sick in the population is sufficiently high. It is clear to see from Figure 3.1 that the expected aggregate social welfare for the subgame with production subsidy of government is smaller than its counterpart for the subgame without government intervention when r_t is low. However, the expected value of total social welfare decreases with r_t at a lower speed for the subgame with government funding than it does for the subgame without government intervention given $p_t(r_t, c_1, w, \theta; a) + \tau < p_t(r_t, c_1)$ (As shown in Appendix B.3). Thus, the expected aggregate social welfare with and without government subsidy would be equivalent at $r_t = r'$. For $r_t > r'$, $E_{c_1}E_\theta\left\{\int_0^T e^{-rt}[r_t(V_1(p_t(r_t, c_1, w, \theta; a) + \tau) + V_1(\tau)) + (1 - r_t)V_2(\tau)]dt\right\} > E_{c_1}\left\{\int_0^T e^{-rt}[r_t V_1(p_t(r_t, c_1)) + (1 - r_t)V_2]dt\right\}$. Consequently, the government would better to involve when the proportion of sick exceeds a certain threshold level r'. For $r_t < r'$, the problem of cost reduction could be left to the monopolist. In addition, a high expected value of the ex-ante production cost c_1 induces a high expected price for treatment and thus a low expected total social welfare for the subgame without government intervention. It follows that if the productivity type of the firm θ was expected to be high, the government involvement by offering production subsidy could be considered to be more effective for reducing

Figure 3.1: Impact of the Growth in r_t on the Expected Value of Total Social Welfare Subgame with Government Fund v.s Subgame without Government Fund



production cost of the drug. Therefore, the government would be more likely to be involved by offering the production subsidy to the foreign drug monopolist when the expected values of the production cost and/or the type parameter of the firm are high and the number of sick in the population reaches a sufficiently large value.

Given the equilibrium outcomes, we proceed with comparative statics to examine the impacts of the prevalence of the disease, the ex-ante production cost of the firm and the parameter of productivity type of the firm on the optimal values of the dynamic prices for treatment and the government production subsidy.

3.2.3 COMPARATIVE STATICS

Given equation 3.5, the function of equilibrium dynamic prices for treatment when the firm accepts the production subsidy of government (i.e., when $\theta \geq \frac{r}{1-e^{-rT}}$), the impact of proportion of sick r_t on $p_t(r_t, c_1, w, \theta; a)$ can be first derived as: $\frac{dp_t(r_t, c_1, w, \theta; a)}{dr_t} = -\frac{1}{2}\alpha\mu_2(t)$ which is negative given $\alpha > 0$ and $\mu_2(t) > 0$. Therefore, the lower the prevalence of the disease, the higher is the price for treatment. It can be explained intuitively as follows: initially, when the proportion of sick is small, the monopolist prices high, so that the patients with the low taste for health (i.e.,those with $\beta < p_t + \tau$) do not buy the treatment and thus increase the chance that those with the high taste for health will be infected subsequently. Consequently, with the increase in the prevalence of the disease, the price for treatment declines associated with the negative externality of treatment-from the monopolist's perspective-on its future market. Second, it can be shown that $\frac{dp_t(r_t, c_1, w, \theta; a)}{d\theta} = -\frac{1}{2}w < 0$ and $\frac{dp_t(r_t, c_1, w, \theta; a)}{dw} = -\frac{1}{2}(\theta + \frac{r}{(1-e^{-rT})}) < 0$ given $w > 0$ for $\theta \geq \frac{r}{1-e^{-rT}}$. The optimal dynamic prices are lower when the type parameter of the firm θ and/or the government subsidy w are higher. As a result, the subsidy fund towards reducing the production cost of the drug would be more effective for the firm with a higher productivity type. Last, the dynamic prices increase with the ex-ante cost of production of the firm: $\frac{dp_t(r_t, c_1, w, \theta; a)}{dc_1} = \frac{1}{2} > 0$. Similarly, given the function of $p_t(r_t, c_1)$, the dynamic prices for treatment in equilibrium when the government decides not to involve (equation 3.20), it can be shown that $\frac{dp_t(r_t, c_1)}{dr_t} = -\frac{1}{2}\alpha\mu_3(t) < 0$ given $\alpha > 0$ and $\mu_3(t) > 0$ and $\frac{dp_t(r_t, c_1)}{dc_1} = \frac{1}{2} > 0$. The optimal dynamic prices $p_t(r_t, c_1)$ also decrease with r_t and increase with c_1 which is consistent with the result as shown for $p_t(r_t, c_1, w, \theta; a)$.

The optimal value of government production subsidy w is greater than zero when $\theta \geq \frac{r}{1-e^{-rT}}$. Additionally, the function of w in equilibrium (i.e., for $\theta \geq \frac{r}{1-e^{-rT}}$) is represented by equation 3.12. First, the impact of the growth in proportion of sick on the value of government funding is derived as follows:

$$\text{(3.22)} \quad \frac{dw}{dr_t} = \frac{-\dfrac{(\overline{\theta}+\underline{\theta})}{8}\alpha\left[\mu_2(t)+2\mu_1(t)\right] - \dfrac{r\alpha}{4(1-e^{-rT})}\left(\mu_2(t)-2\mu_1(t)\right) - \dfrac{1}{r_t^2}\dfrac{r}{(1-e^{-rT})}}{\dfrac{1}{12}(\overline{\theta}^2+\overline{\theta}\underline{\theta}+\underline{\theta}^2) + \dfrac{r(\underline{\theta}+\overline{\theta})}{4(1-e^{-rT})} - \dfrac{3r^2}{4(1-e^{-rT})^2}}$$

The denominator is positive given $w > 0$. As $\mu_2(t) > 0$, $\mu_1(t) < 0$ and $\mu_2(t) + 2\mu_1(t) < 0$, clearly, $\frac{dw}{dr_t} > 0$ if $-\frac{(\overline{\theta}+\underline{\theta})}{8}\alpha\left[\mu_2(t)+2\mu_1(t)\right] > \frac{r\alpha}{4(1-e^{-rT})}\left(\mu_2(t)-2\mu_1(t)\right) + \frac{1}{r_t^2}\frac{r}{(1-e^{-rT})}$ and vice versa. $-\frac{(\overline{\theta}+\underline{\theta})}{8}\alpha\left[\mu_2(t)+2\mu_1(t)\right]$ measures the utility gain of consumers due to the decline in prices for treatment associated with the government production subsidy while $\frac{r\alpha}{4(1-e^{-rT})}\left(\mu_2(t)-2\mu_1(t)\right)+\frac{1}{r_t^2}\frac{r}{(1-e^{-rT})}$ measures the utility loss of consumers for paying taxes to compensate the production subsidy of government. It can be explained intuitively as follows: the government would like to raise the level of subsidy fund with the growth in proportion of sick if the cost deduction associated with the government funding would benefit consumers a lot. In contrast, if the government intervention could not make consumers obtain sufficient utility gains but make them worse off by paying for the taxes, the level of production subsidy of government would be reduced with r_t. It should be noted that the lump-sum tax is paid by every consumer whether they have the disease or not. Therefore, when the prevalence of the disease is high, it means there is a greater probability of matching the payment for treatment with expected benefits from government involvement. As a result, the government would better to involve when the proportion of sick in the population is sufficiently large.

The type parameter of the firm is assumed to be uniformly distributed on the interval $[\underline{\theta}, \overline{\theta}]$. Thus the expected value of θ is calculated as $E(\theta) = \frac{(\underline{\theta}+\overline{\theta})}{2}$. Next, we derive the impact of expected value of firm's productivity type on the level of government funding. The derivative of w with respect to $\overline{\theta}$ is derived from equation 3.12 as:

$$\text{(3.23)} \qquad \frac{dw}{d\overline{\theta}} = \frac{\frac{1}{8}\left\{\frac{(\overline{c}+\underline{c})}{2} - 1 + \alpha(1-r_t)(\mu_2(t)+2\mu_1(t))\right\} C_1 - \left[\frac{(2\overline{\theta}+\underline{\theta})}{12} + \frac{r}{4(1-e^{-eT})}\right] C_2}{C_1^2}$$

where $C_1 = \frac{1}{12}(\overline{\theta}^2 + \overline{\theta}\underline{\theta} + \underline{\theta}^2) + \frac{r(\underline{\theta}+\overline{\theta})}{4(1-e^{-rT})} - \frac{3r^2}{4(1-e^{-rT})^2}$ and C_2 is the numerator of equation 3.12. As $0 < \underline{c} < \overline{c} < 1$ and $\mu_2(t) + 2\mu_1(t) < 0$, $\frac{(\overline{c}+\underline{c})}{2} - 1 + \alpha(1-r_t)(\mu_2(t)+2\mu_1(t)) < 0$. And $\frac{(2\overline{\theta}+\underline{\theta})}{12} + \frac{r}{4(1-e^{-rT})} > 0$ since $0 < \underline{\theta} < \overline{\theta} < 1$. Given $w > 0$, C_1 and C_2 are both positive. As a result, $\frac{dw}{d\overline{\theta}} < 0$. Given $\underline{\theta}$, the higher the $\overline{\theta}$, the higher is the expected value of θ. Hence, the optimal level of government production subsidy is low when the productivity type of the firm is expected to be high. It could be explained as follows: as the cost of production is defined as $c(w, \theta) = c_1 - \theta w$, when the type parameter of the firm is expected to be high, even the small amount of government subsidy could reduce the production cost of the firm effectively. Therefore, the corresponding government funding declines. Given $c_1 \sim U(\underline{c}, \overline{c})$, the expected value of c_1 is equal to $\frac{(\underline{c}+\overline{c})}{2}$. Therefore, the impact of $E(c_1)$ on the value of government fund is derived from equation 3.12 as:

$$\text{(3.24)} \qquad \frac{dw}{dE(c_1)} = \frac{\frac{(\overline{\theta}+\underline{\theta})}{8} + \frac{r}{4(1-e^{-rT})}}{C_1}$$

Obviously, $\frac{dw}{dE(c_1)} > 0$ given $0 < \underline{\theta} < \overline{\theta} < 1$ and $C_1 > 0$ for positive w so the higher level of $E(c_1)$ induces the more amount of production subsidy from the government. It is easy to be explained as follows: if it was expected that the ex-ante marginal cost of production was very high, the government would have to provide more funding to the firm to reduce the cost sufficiently ex-post.

3.3 EMPIRICAL APPLICATION

The empirical test for the theoretical model is conducted in this section. The government's and the monopolist's dynamic problems are solved by using a newly developed dynamic optimization toolbox, dynopt, of matlab (Matlab dynamic optimization code-dynopt; User's Guide). The data used in analysis are shown in Table 3.1.

Table 3.1: Data Used in Analysis for Empirical Application

Parameters	r	α	$\underline{c}$	$\bar{c}$	$\underline{\theta}$	$\bar{\theta}$	t_0	$t_f(T)$
Values	0.05	2	0.6(0.8)	1	0.08(0.8)	1	0	20

Optimal problem of the government defined by Equation 3.10 is first solved given $\underline{c} = 0.6$, $\bar{c} = 1$, $\underline{\theta} = 0.08$ and $\bar{\theta} = 1$. Second, the same problem is resolved with different value of $\underline{c}$ and $\underline{\theta}$ respectively to compare the optimal levels of production subsidy w. Third, for every case, with the resulting optimal values of w, the time paths of expected price for treatment and expected social welfare are obtained through equations 3.5 and 3.10. For the subgame without government intervention, the expected price path and the expected proportion of sick path are obtained by solving the problem defined by equation 3.19 given $E(c_1)$. With the dynamics paths of the expected social welfare, the speed of convergence of the expected value of social welfare with government subsidy to its counterpart without government intervention could be compared for the cases with different expected values of c_1 and θ then. The empirical results are shown in Table 3.2 to Table 3.4. Figure 3.2 to Figure 3.5 show the dynamics of expected value of social welfare with proportion of sick, the time paths of expected social welfare, proportion of sick and price for treatment for the subgame with production subsidy of government versus the subgame without government involvement. Figure 3.6 to Figure 3.9 and Figure 3.10 to Figure 3.13 show the same dynamics paths when the expected value of ex-ante cost of production and the expected type parameter of the firm are raised, respectively.

3.3.1 EMPIRICAL RESULTS

The empirical results of the optimization problem of government are shown in Table 3.2. When $\underline{c} = 0.6$, $\bar{c} = 1$, $\underline{\theta} = 0.08$ and $\bar{\theta} = 1$, the optimal level of government subsidy w is equal to 0.1165 and the corresponding tax rate is 0.0092 given $r = 0.05$ and $T = 20$. While in case 2 and case 3, the value of government funding increases to 0.344 and decreases to 0.0675 respectively with the increase in $\underline{c}$ and $\underline{\theta}$. Therefore, the government needs to offer more production subsidy when the ex-ante cost of production is expected to be high. In contrast, the level of government funding is lower when the type parameter of the firm is expected to be high. Apparently, the empirical results are consistent with the predictions of the theoretical model implied by equations 3.23 and 3.24.

Table 3.2: Optimal Levels of Government Production Subsidy with Different Expected Values of Ex-ante Production Cost and Type Parameter of the Firm

Variable	Case 1	Case 2: raise E(c_1)	Case 3: raise E(θ)
$\underline{c}$	0.6	0.8	0.6
$\bar{c}$	1	1	1
$\underline{\theta}$	0.08	0.08	0.8
$\bar{\theta}$	1	1	1
w	0.1165	0.344	0.0675
τ	0.0092	0.0272	0.0053

The starting point of the proportion of sick, r_{t_0}, is set to be at 0.1. As shown in Figure 3.4 and Figure 3.5, Figure 3.8 and Figure 3.9 and Figure 3.12 and Figure 3.13, for both subgames with and without government production subsidy, the expected proportion of sick increases with time first and then remains constant. Correspondingly, the expected price for treatment declines first and then remains roughly constant as well. It follows that, as predicted by the theoretical model, the monopoly price for treatment declines with the number of sick in the population. Additionally, the empirical results also show that the monopoly price and prevalence of the disease paths are expected to converge to a non-zero steady state which is consistent with the result derived by S.Mechoulan (2007). Table 3.3 shows that for every case, the prices for treatment are expected to be reduced sufficiently by accepting the government subsidy (i.e., $E(p_1(t)+\tau) < E(p_2(t))$). This implies that the proportion of sick grows slower with government funding. Figure 3.2, Figure 3.6 and Figure 3.10 show that the initial value of expected social welfare for the subgame with government production subsidy is smaller than its counterpart for the subgame without government intervention. However, with the growth in r_t, the expected social welfare with subsidy $W_1(t)$ decreases at a lower speed and reaches the expected value of social welfare without government fund $W_2(t)$ at $r_t = r'_i$ at time $t = t'_i$, $i = 1,2,3$. For $r_t > r'_i$, $i = 1,2,3$ (i.e., $t > t'_i$, $i = 1,2,3$), $W_1(t) > W_2(t)$. Thus, government intervention would benefit consumers more when proportion of sick in the population is large enough. Clearly, all empirical results are consistent with that implied in the theoretical model. Compared with other cases, when the ex-ante cost of production is expected to be high (case 2), the price for treatment is reduced mostly with government fund and therefore the proportion of sick grows slower. It is due to the fact that government offers more production subsidy when the expected value of production cost of the firm is high.

Table 3.3: Expected Starting and Termination Values of Proportion of Sick, Price for Treatment and Social Welfare with and without Production Subsidy for Different Cases

Variable	Case 1		Case 2		Case 3	
	With Fund	Without Fund	With Fund	Without Fund	With Fund	Without Fund
r_{t_0}	0.1	0.1	0.1	0.1	0.1	0.1
r_{t_f}	0.43208	0.44636	0.43056	0.47381	0.43151	0.44636
$p_{t_0}(p_{t_0}+\tau)$	0.95396(0.96316)	0.94079	0.93359(0.96079)	0.95567	0.95696(0.96226)	0.94079
$p_{t_f}(p_{t_f}+\tau)$	0.86396(0.87316)	0.90364	0.84359(0.87079)	0.97129	0.86696(0.87226)	0.90364
W_{t_0}	0.44107	0.45034	0.42312	0.45014	0.44492	0.45034
W_{t_f}	0.10283	0.10282	0.0969	0.0969	0.10427	0.10282

Table 3.4 shows the impact of expected values of ex-ante cost of production and type parameter of the firm on the speed of convergence of the expected social welfare with government production subsidy to its counterpart without government intervention. With a higher $E(c_1)$, the level of production subsidy of government raises and thus the prices for treatment are reduced more. Whereas with a higher $E(\theta)$, government involvement by offering subsidy fund would be more efficient and the taxes paid by the consumers become less at the mean while. As a result, $W_1(t)$ meets the value of $W_2(t)$ earlier for case 2 and case 3 (i.e., $t'_1 > t'_2 > t'_3$). Thus the government involvement may be considered to be more effective when the cost of production and/or the type parameter of the firm are expected to be high which is consistent with the theoretical prediction. Compared with other cases, when the expected type parameter of the firm is high (case 3), $W_1(t)$ converges to $W_2(t)$ with the highest speed. Hence, $W_1(t)$ hits $W_2(t)$ at the lowest r_t and the consumers end up with the highest utilities as well. It is associated with the sufficiently lower price for treatment by accepting the government fund and the fairly low tax rate at the meanwhile. Also, note that when the expected ex-ante cost of production is raised (case 2), the level of government funding arises but the tax rate paid by consumers also increases. As the disutility associated with the taxes is high at a low level of r_t, the total utility of consumers for the subgame with government production subsidy is much lower than its counterpart for the subgame without the funding at the initial value of $r_t = r_{t_0}$. As a result, $W_1(t)$ hits $W_2(t)$ at a later time (i.e., at a higher level of r_t) than it does in case 3 although the prices for treatment are reduced more in case 2.

Table 3.4: Comparison of Speed of Convergence of Expected Social Welfare with and without Government Subsidy for Different Cases

Case 1					
r'_1	t'_1	$W_1(t'_1)$	$W_2(t'_1)$	$p_1(t'_1)$	$p_2(t'_1)$
0.42675	6	0.2096	0.2096	0.8787	0.9060
Case 2					
r'_2	t'_2	$W_1(t'_2)$	$W_2(t'_2)$	$p_1(t'_2)$	$p_2(t'_2)$
0.40711	4.65	0.2181	0.2181	0.8595	0.9514
Case 3					
r'_3	t'_3	$W_1(t'_3)$	$W_2(t'_3)$	$p_1(t'_3)$	$p_2(t'_3)$
0.3697	3.45	0.26328	0.26328	0.8856	0.9082

3.4 CONCLUSION

The paper finds that government funding can be utilized to improve welfare when a monopolist supplies a needed pharmaceutical drug aimed at curtailing the speed of a communicable disease. Additionally, the paper characterizes the choice of government funding policy in the context of communicable diseases where market power and heterogenous agents are present. The economic choice of consumers for treatments influences the spread of the disease. The local government chooses whether or not to offer a production subsidy for reducing the production cost of treatment to a foreign drug monopolist by maximizing the expected value of total social welfare. If the government decides to provide the offer, the firm could either accept or reject it. The ex-post cost of the firm by receiving the government funding depends on the productivity type parameter of the firm θ and the production subsidy w. If the firm accepts the government subsidy, the dynamic prices for treatment are determined by the monopolist by maximizing the aggregate profit subject to the prevalence path of the disease given the ex-post cost of production. While if the firm rejects the offer or if the government decides not to provide the production subsidy in the first stage, the market prices for treatment are determined dynamically by the monopolist producing at the ex-ante cost c_1.

The key findings of the study are as follows: First, the foreign drug monopolist accepts the production subsidy of local government if its productivity type is sufficiently high (i.e., $\theta \geq \frac{r}{(1-e^{-rT})}$). Additionally, the monopoly dynamic prices for treatment decline with the prevalence of the disease. Second, when the optimal level of government subsidy is greater than zero, the value of funding increases with the expected ex-ante cost of production and decreases with the expected productivity type of the firm. Further, the government would like to raise the level of subsidy fund with the growth in proportion of sick if the decrease in price for treatment associated with the government fund would benefit consumers sufficiently. Finally, the local government would be more likely to be involved by offering production subsidy to the foreign drug monopolist when the ex-ante cost of production and/or the productivity type parameter of the firm are expected to be high and the number of sick in the population reaches a sufficiently large value. In empirical application, the optimal dynamic problems defined by the theoretical model are solved by using the newly developed dynamic optimization toolbox, dynopt, of matlab. The empirical results are consistent with the predictions of the theoretical framework.

The extension of the previous work by Mechoulan (2007) in the present paper is accomplished through the introduction of the government and the monopolist's choice whether or not to accept the government subsidy aimed at reducing the cost of production of the pharmaceutical drug. The paper examines the efficiency of government production subsidy in reducing monopoly prices for treatment, lowering the spread of the disease and raising the level of total social welfare. The paper also demonstrates the theoretical results through the application of a computational dynamic optimization model. The paper contributes to the literature regarding the economics of pharmaceutical drug production for communicable diseases in an economy with market power, externalities and heterogenous agents by introducing a government with the capacity to tax and subsidize the local production of the drugs aimed at reducing the production cost of the drugs and thus lowering the local prices for treatments. The influence of government funding on social welfare improvement through subsidizing a potential entrant to reduce its fixed entry cost as opposed to fund the incumbent drug monopoly firm is also formally considered. The research efforts towards this perspective are summarized by another paper but the theoretical framework is shown in Appendix B.4.

Figure 3. 2: Dynamics of Expected Social Welfare with r_t: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

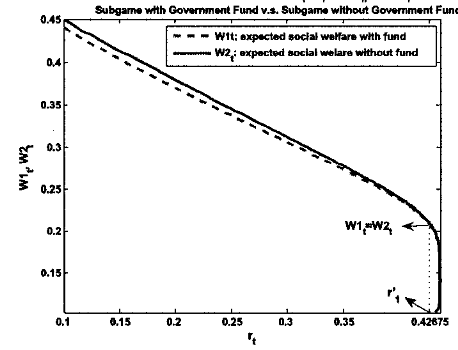


Figure 3.3: Expected Social Welfare Path: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

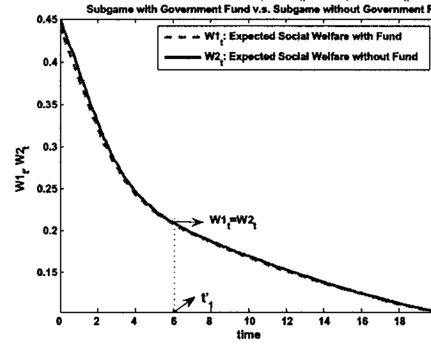


Figure 3.4: Expected Proportion of Sick Path: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

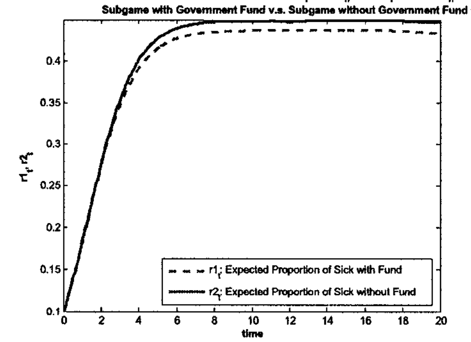


Figure 3.5: Expected Price for Treatment Path: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s Subgame without Government Fund

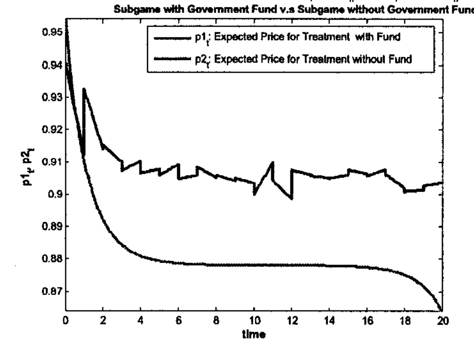


Figure 3.6: Dynamics of Expected Socal Welfare with r_t: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

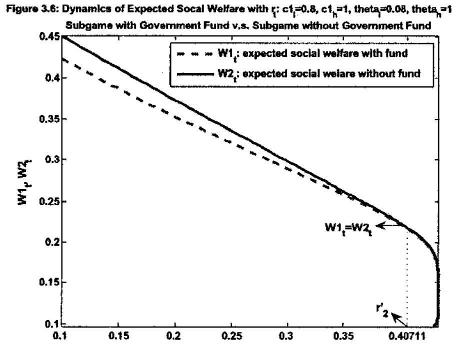


Figure 3.7: Expected Social Welfare Path: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

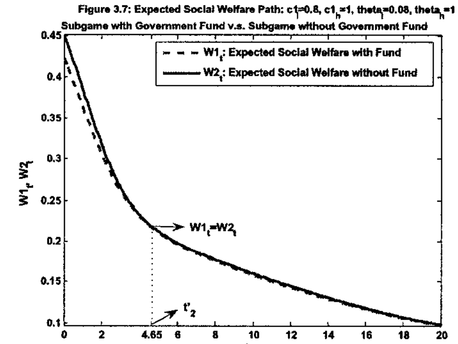


Figure 3.8: Expected Proportion of Sick Path: $c1_l$=0.8, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

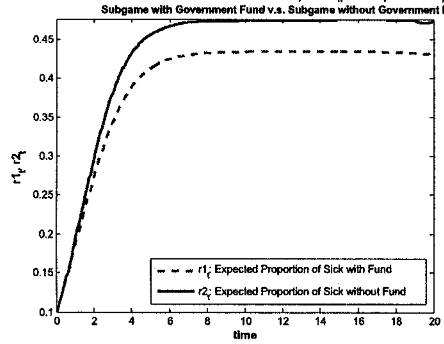


Figure 3.9: Expected Price for Treatment Path: $c1_l$=0.8, $c1_h$=1, $theta_l$=0.08, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

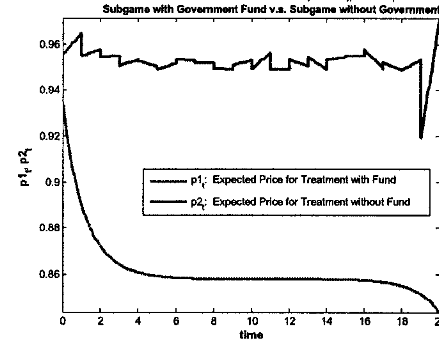


Figure 3.10: Dynamics of Expected Social Welfare with r_t: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.8, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

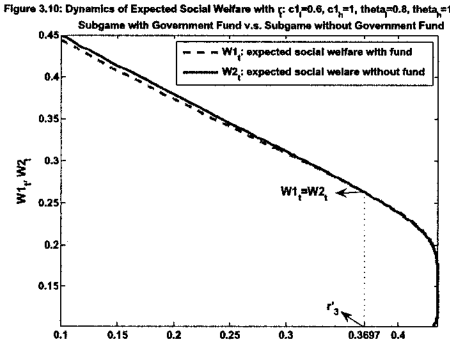
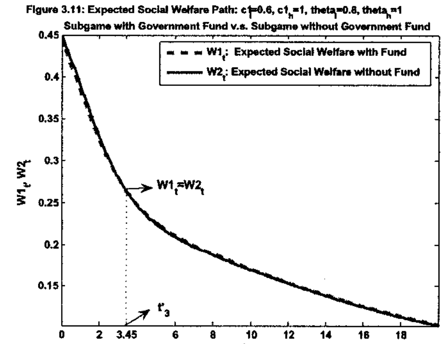


Figure 3.11: Expected Social Welfare Path: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.8, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund



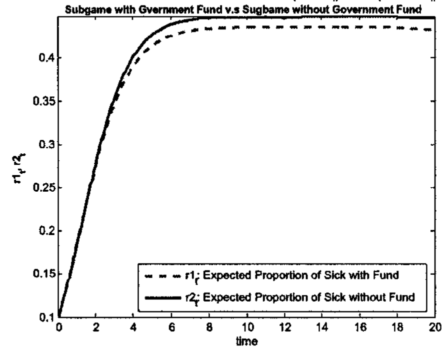


Figure 3.12: Expected Proportion of Sick Path: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.8, $theta_h$=1
Subgame with Gvernment Fund v.s Sugbame without Government Fund

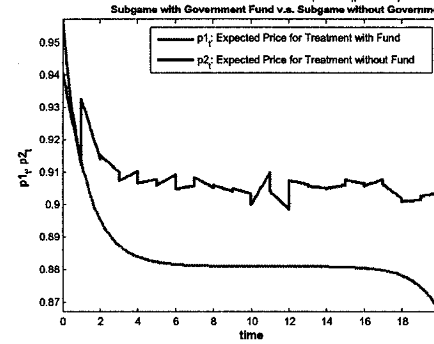


Figure 3.13: Expected Price for Treatment Path: $c1_l$=0.6, $c1_h$=1, $theta_l$=0.8, $theta_h$=1
Subgame with Government Fund v.s. Subgame without Government Fund

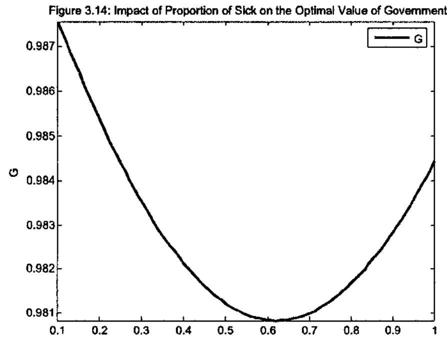


Figure 3.14: Impact of Proportion of Sick on the Optimal Value of Government Fund

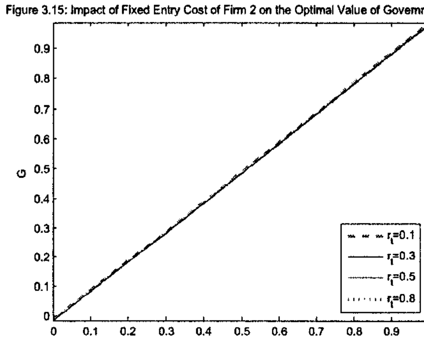
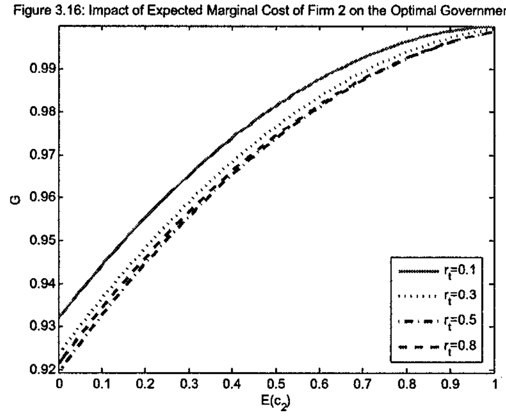


Figure 3.15: Impact of Fixed Entry Cost of Firm 2 on the Optimal Value of Government Fund

Figure 3.16: Impact of Expected Marginal Cost of Firm 2 on the Optimal Government Fund



4. SEARCH INTENSITY, JOB OFFER ARRIVAL RATE AND LABOR MARKET TRANSITIONS

4.1 INTRODUCTION

In this paper, we will focus on the influence of job search behavior, or more specifically, the influence of search intensity of individuals who are unemployed on the job offer arrival rate and thus on the labor market transitions in a stationary framework using the Canadian data. [21] The original (standard) wage search model models the behavior of job seekers as a sequential search process in which both the job offers received per period and the wage offers arrive randomly with constant and certain rates of arrival known to the unemployed workers. At each period, taking the cost of search into account, the unemployed decide whether or not to stop searching by accepting the best wage offer received during that period or to continue to search to maximize the expected value of future net income. The optimal strategy is characterized by the choice of reservation wage, and the model generates implications for the distribution of unemployment spell and the wage received after a transition into employment. Mortensen (1986) extends the standard wage search model by endogenizing the search effort to model the behavior of on the job search. In the model, the job offer arrival rate is assumed to be proportional to the worker's "search effort" and the cost of search is assumed to be an increasing convex function of "search effort". With endogenous search effort, the optimal strategy is a choice of the reservation wage and the intensity of search for both job seekers who are unemployed and who are currently employed. Further, the optimal intensity of search equates the marginal returns to search and the marginal cost of search. Another extension of the standard search model is to endogenize the wage distribution, by

[21] The optimal strategy of job seekers in a non-stationary job search model is also considered. However, due to the scope of this paper, the details of the non-stationary model are deferred to be shown in another paper. But the equations characterizing the optimal search intensity and optimal reservation wage in a non-stationary framework are shown in the Appendix C.3.

incorporating equilibrium implications (see Burdett and Mortensen 1998; Ridder and Van den Berg 1998). The present paper focuses on the first extension of the original model in which the job offer arrival rate is endogenized via search effort.

In empirical work, most studies on individual labor market transitions pay a little attention to the search process leading to job offers. To our knowledge, there are only a few empirical studies in which the impact of search behavior on the job offer arrival rate and on the unemployment duration are analyzed. Yoon (1981) investigates the role of search time for an offer in the determination of unemployment duration. Lindeboom and Theeuwes (1993) pay special attention to the effect of the benefit level on the number of search contacts in a simultaneous analysis of unemployment duration and search intensity. Koning, Van den Berg and Ridder (1997) estimate a structural search model, which distinguishes the effects between formal (applications) and informal (referrals) search methods on job offer arrival rate and on the subsequent wages. Bloemen (2005) studies the influence of job search on labor market transitions for both the unemployed job seekers and those searching on-the-job in an empirical structural job search model.

Furthermore, the literature on the process of job search is largely confined to US, British and Dutch studies. By contrast, the empirical work for Canada is very limited. A notable exception is Lars Osberg (1993) who examines the job-search methods of jobless workers and emphasizes sample selectivity in choice of job-search strategies (especially use of public employment agencies). Empirical tests using longitudinal data from the Labor Force Survey of Canada for 1981, 1983 and 1986 indicate that job-search methods change with the business cycle.

So far, the incentive effect of benefit sanctions has not attracted much attention in the literature. Yet a study by Grubb(1999) shows that sanctions are now an important policy tool in many OECD countries. The empirical evidence is mixed. For instance, for the

Netherlands, no effect of benefit cuts is found in most studies (Van den Berg(1990b)). However, after correction for selectivity in the imposition of sanctions, Abbring, Van den Berg and Van Ours(2000) find that benefit sanctions raise individual re-employment rate substantially. But it should be noted that the sanction examined in their study is a temporary and limited benefit cut. A harder sanction policy (i.e., a permanent suppression of UI benefits if two job offers are refused) is simulated by Stéfan Lollivier and Laurence Rioux (2002) using the French sample of the European Survey(1994-1998). They find out that the expected duration of unemployment seems to be shortened sufficiently by implementing a sanction policy.

Investigating the impact of search intensity of individuals who are unemployed on the transition rate into employment by influencing the job offer arrival rate using the data of Labor Market Activity Survey (LMAS) of Canada, 1988-1990, is the focus of this present study. An empirical model of job search with endogenous search intensity is estimated in a stationary framework, based on the model by Mortensen (1986). In Mortensen's (1986) model, the search effort is modeled as a one-dimensional variable. However, there exist several indicators of search in the Canadian LMAS data. As a result, three indicators concerning the job search behavior; one for "search or not", two for "search intensity" are constructed using the information provided by the data. The estimation results show that three indicators of search do influence the job offer arrival rate significantly and compared with the others, the number of channels used for search has the largest effects. In addition, the exogenous part of the arrival rate of job offers which depends on the characteristics of individuals is higher for those who are highly educated, more professional and living in the industrialized area. Thus, the unemployed workers who have more market opportunities search more intensively due to their higher value of return to search. The empirical results also reveal that the higher search intensity accelerates the labor market transitions effectively. The unemployed with a higher level of search effort and a relatively lower value in reservation wage transit into work sooner compared with the others. To evaluate the incentive effects of the unemployment benefits on the behavior of job search and thus on the transition rate into employment, we simulate two alternative reforms on current unemployment insurance compensation system which produce the same benefit saving for the government. The simulation results show that the deduction in insurance benefits do raise the level of search effort and reduce the value of reservation wage for the unemployed at the same time. Compared with the permanent benefit cut, the monitoring and benefit sanction for insufficient search seems to be a better way to provide incentives for the unemployed workers to search more and demand less to get back to work soon. However, the simulation results also reveal that the policy changes on insurance compensation scheme are less effective for those who lack of market opportunities due to their bad personal characteristics. To them, offering more programs of education and training courses for specific skills may be a more feasible policy tool.

The paper is organized as follows. Section 2 presents the economic model. The empirical implementation of the structural model is proposed in section 3 and the data are described in section 4. While in section 5, the estimation results of the empirical model are presented. The two alternative reforms of the unemployment compensation scheme are simulated and their effects on the search behavior and labor market transitions are assessed in section 6. In the final section, we present the conclusion.

4.2 THE MODEL

Consider a structural model which is stationary based on Mortensen's standard job search model (1986). The original model by Mortensen (1986) describes both the search behavior of the unemployed workers and on the job search. The model assumes that wage offers arrive randomly from a wage offer distribution, $F(w)$, and individuals maximize the expected present value of future net wealth. The offer arrival rate is proportional to the worker's "search effort" and that the cost of search is a increasing convex function of "effort". In the present study, we focus on the search behavior of the individuals who are unemployed only. It is assumed that search effort s is defined as $s = (s_1, s_2, s_3)$ where s_j represents the search indicator by channel j, $j = 1, 2, 3$. s_1 can be interpreted as the search attitude measurement indicating whether or not an unemployed worker did search a job. s_2 and s_3 measure the number of methods used for job search and the amount of time spent on search by an unemployed respectively.

Next, we express the job offer arrival rate for individual i who is unemployed, $m_i(s)$, as the product of the market-determined part of the arrival rate of job offers, λ_i, and a composite sum of various indicators of search intensity, corresponding to its theoretical counterpart, $\lambda_i s$, in Mortensen's model: $m_i(s) = [\alpha_0 + \alpha_1 s_1 + s_1(\alpha_2 s_2 + \alpha_3 s_3)]\lambda_i$, with $\alpha_j > 0, j = 0, 1, 2, 3$ and $\lambda_i > 0$. To simplify the expression for $m_i(s)$, we can rewrite it in a vector form as follows: $m_i(s) = (\alpha_0 + \alpha S')\lambda_i$. Search effort is indicated by S, $S \geq 0$, and S is allowed to be a vector of search indicators: $S = (s_1, s_1 s_2, s_1 s_3)$. Search effectiveness parameter $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ measures the impact of search intensity on the arrival rate which is a vector of equal dimension as S. The person who report not to be searching may get a job due to the reason like personal contacts so the parameter α_0 is identified to capture the transitions into work of non-searchers. The functional form we applied here for arrival rate of job offers simply implies two issues. First, the zero value of s_1 implies that both the values of s_2 and s_3 are zero as well. It means when the unemployed individuals do not search at all, $s_1 = 0$, they do not exert any effort on search, $s_2 = s_3 = 0$, and therefore, $S = 0$. As a result, their job offer arrival rate only depends on the parameter α_0 and the market-determined part of the job offer arrival rate λ: $m(0) = \alpha_0 \lambda$. Second, given the individuals do search jobs, $s_1 > 0$, and hence, $S > 0$, higher search effort increases the probability of receiving a job offer, $dm(s)/dS > 0$.

As in the Mortensen's model(1986), the benefit level for an unemployed worker is assumed to be a constant over the spell of unemployment and is denoted by b. The exogenous part of the job offer arrival rate, λ_i, which is determined by the characteristics of individual i is also assumed to be constant. The wage offers arrive randomly from a wage offer distribution, $F(w)$, which is assumed to be unchanging over time and known to the worker. In addition, the analysis is restricted to the case of no recall of offers received in previous periods for simplicity. The cost of search function is denoted as $c(s)$ which is a increasing and convex function of its argument. In other words, $c(s)$ is convex, having the property $c'(s) > 0$ where the marginal cost of search $c'(s)$ is of equal dimension as s. Moreover, since we have three different indicators of search intensity corresponding to three search channels observable in the data, it is simple to assume that the cost of search function is additively separable in search channels: $c(s) = c_0 + \sum_{j=1}^{3} c_j(s_j)$, where c_0 represents the fixed cost of search and $c_j(s_j)$ denotes the cost of search function by channel j, $j = 1, 2, 3$. Further, let $W(w)$ represent the given present value of stopping, accepting the best offer received, w, during any period and keeping that job forever after at wage w, and V the value of continued search. It is assumed that the individuals who are unemployed maximize the expected discounted value of future net income, V:

$$\rho V = \max_{(s_1, s_2, s_3)} \left\{ b - [c_0 + \sum_{j=1}^{3} c_j(s_j)] + [\alpha_0 + \alpha_1 s_1 + s_1(\alpha_2 s_2 + \alpha_3 s_3)]\lambda \int_0^\infty \{\max[V, W(x)] - V\}\, dF(x) \right\} \tag{4.1}$$

where $\rho > 0$ represents the constant discount factor. The solution of the maximization problem is characterized by the optimal intensity of search $s^* = (s_1^*, s_2^*, s_3^*)$ and a reservation wage ξ. For a stationary job search model, the optimal strategy satisfies the reservation property which implies that the reservation wage, ξ, is defined as a unique solution to $W(\xi) = V$.

The present value of a future earning stream given a wage equal to x is assumed to be $W(x) = x/\rho$. And together with the reservation property, the following is derived from Equation 4.1:

$$(4.2) \qquad \xi = b - [c_0 + \sum_{j=1}^{3} c_j(s_j^*)] + [\alpha_0 + \alpha_1 s_1^* + s_1^*(\alpha_2 s_2^* + \alpha_3 s_3^*)]\, \lambda \int_{\xi}^{\infty} [W(x) - V]\, dF(x)$$

where s_j^* denotes the optimal search intensity of unemployed job seekers by channel $j, j = 1, 2, 3$. Let $\bar{s}_j$ represent the latent search intensity for which the marginal cost of search by means of channel j is equal to the marginal returns to search. The first order condition for the search intensity choice problem on the RHS. of Equation 4.2 yields:

$$(4.3) \qquad c_1'(\bar{s}_1) = [\alpha_1 + \alpha_2 \bar{s}_2 + \alpha_3 \bar{s}_3]\lambda \int_{\xi}^{\infty} [W(x) - V]\, dF(x) \;\; and$$

$$(4.4) \qquad R_1 = c_1'(\bar{s}_1) = [\alpha_1 + \alpha_2 \bar{s}_2 + \alpha_3 \bar{s}_3]\lambda \int_{\xi}^{\infty} [W(x) - V]\, dF(x)$$

And

$$(4.5) \qquad c_j'(\bar{s}_j) = \alpha_j \bar{s}_1 \lambda \int_{\xi}^{\infty} [W(x) - V]\, dF(x) \quad \forall\, j = 2, 3 \;\; and$$

$$(4.6) \qquad R_j = c_j'(\bar{s}_j) = \alpha_j \bar{s}_1 \lambda \int_{\xi}^{\infty} [W(x) - V]\, dF(x) \quad \forall\, j = 2, 3$$

in which R_j denotes the marginal returns to search by channel j and α_j can be interpreted as the parameter indicating the search effectiveness of search channel j, $j = 1, 2, 3$. Then the optimal level of search intensity s_j^* equals $\max[0, \bar{s}_j]$. Thus, the optimal search intensity satisfies the marginal cost of search equals the marginal returns to search condition if $s_j^* > 0$. Apparently, Equations 4.2, 4.3 and 4.5 are used to determine the optimal strategy ξ and $\bar{s}_j$

simultaneously, $j = 1, 2, 3$. We notice that Equations 4.2, 4.3 and 4.5 reveal some important properties of the optimal search intensity. First, by the convexity of the cost of search function ($c''(s) > 0$), the marginal cost of search is increasing in search intensity. This implies that given the unemployed workers did search a job, ($\bar{s_1} > 0$), if the effectiveness of search by channel j, represented by the factor $\alpha_j \lambda$, rises, the optimal intensity of search by channel j rises as well, $j = 2, 3$ (Equation 4.5). This also applies to the decision to search: a low value of the effectiveness of search, $\alpha_1 \lambda$, may induce unemployed workers not to search (Equation 4.3). Second, Equations 4.2, 4.3 and 4.5 describe the simultaneous movement of the optimal search intensity and the reservation wage. Since Equation 4.2 implies that the benefit level and the reservation wage for the unemployed workers are positively correlated, a higher benefit level raises the reservation wage and reduces the intensity of search simultaneously according to Equations 4.2, 4.3 and 4.5. Thus, lowering the level of unemployment insurance may possibly be an effective method for policymakers to induce the unemployed to search more and ask for less reservation wages to exit from unemployment more quickly.

Clearly, the optimal results captured by Equations 4.2, 4.3 and 4.5 are in contrast to those in Mortensen's (1986) since we did some modifications to the original model. First, we add a constant term α_0 to capture the transition into employment of non-searchers. However, in Mortensen's model(1986), it is assumed that a zero search effort implies a zero job offer arrival rate and therefore no transition into employment at all. Second, we define search effort as a vector of three different search indicators like $s = (s_1, s_2, s_3)$ as opposed to a one-dimensional search intensity, s, used in original model. Last, we specify the job offer arrival rate as a function of the composite sum of various indicators of search intensity while in Mortensen's (1986) model, the arrival rate of job offers depends on the single-dimensional overall search effort s only. However, if $\alpha_0 = 0, s_2 = s_3 = 0$ and s_1 represented the total search effort, our model would collapse with the model of Mortensen (1986). Consequently, in that case, the equations characterizing the optimal strategy for an unemployed job seeker across the two models would be equivalent.

Finally, we allow the unobserved heterogeneity of individuals to enter via the market-determined part of the arrival rate, λ. It is assumed that for individual i, $\lambda_i = \exp(X_i'\beta + q_i)$ in which X_i is a vector of individual characteristics, β is a vector of parameters and q_i represents the unobserved heterogeneity which is normally distributed with mean zero and variance 1.

The inferences implied by Equations 4.2, 4.3 and 4.5 will be tested by estimating an empirical model specified in next section. The estimation results will give us some insight in the influence of intensity of search on job offer arrival rate and therefore on the probability of transition into work for unemployed individuals. Moreover, the simultaneous movement of search intensity and reservation wage predicted by the theoretical model will be examined by the simulations of policy changes on insurance compensation system presented in section 4.6. The impacts of changes of unemployment insurance on the search behavior and labor market transitions for the unemployed who are differentiated in personal characteristics will also be compared. Simulation results may shed some light on how to provide incentives for the unemployed workers to search more and demand less to improve job finding success.

4.3 EMPIRICAL IMPLEMENTATION

4.3.1 BASIC SPECIFICATION

First, in order to obtain explicit expression for the optimal intensity of search along various channels, we need to specify the cost of search function. In the literature on search models with endogenous search effort s and a single search channel, the arrival rate and the search costs are generally taken to be proportional to s and s^2, respectively (see the survey by

Mortensen and Pissarides, 1999). However, in present study, we have three different search channels observable in the data. Therefore, in section 4.2, we assume that the cost of search function is additively separable in search channels. As for the specification for the cost of search function by means of channel j, we define $c_j(s_j)$ as a quadratic function of s_j, $j = 1, 2, 3$. Consistent with the theoretical model, we take the following specification: [22]

$$(4.7) \qquad c(s) = c_0 + \sum_{j=1}^{3} c_j(s_j) \;\; with \;\; c_j(s_j) = \frac{1}{2}\, c_j\, s_j^2$$

In which c_0 is the fixed cost of search and c_j are parameters with $0 < c_j < \infty, \;\; j = 1, 2, 3$. Note that $c(s)$ is convex since $c_j > 0, \;\; j = 1, 2, 3$.

As we assumed in section 4.2, the exogenous part of arrival rate of job offers λ depends on the characteristics of individuals. Additionally, the unobserved heterogeneity of individuals is allowed to enter via λ. The market-determined part of job offer arrival rate for individual i, λ_i, is thus specified as:

$$(4.8) \qquad \ln \lambda_i = X_i'\beta + q_i \;\; with \;\; q_i \sim N(0, 1)$$

[22] A general function of cost of search is utilized for the theoretical model. The restricted form of cost function is specified for the empirical model only. In the literature on search models with endogenous search effort s, the search costs are generally taken to be proportional to s^2 as outlined in the survey by Mortensen and Pissarides, 1999). For instance, van den Berg and van der Klaauw (2001), specified the cost of search function with two search channels as a quadratic form as follows: $c(s_1, s_2) = \frac{1}{2}c_0(s_1+s_2)^2$ with $0 < c_0 < \infty$. In addition, Abbring, van den Berg and van Ours (2005), defined the cost of search function as: $c(s) = \frac{1}{2}c_0 s^2$. A similar cost function specification used in the present study. Specifically, the cost of search function is assumed to be additively separable in search channels: $c(s) = c_0 + \sum_{j=1}^{3} c_j(s_j)$. The cost of search function by channel j, $c_j(s_j)$, is defined as a quadratic function of s_j as follows: $c_j(s_j) = \frac{1}{2}c_j s_j^2$, $j = 1, 2, 3$ where $0 < c_j < \infty$. The optimal level of search intensity emerging from the theoretical model is characterized by marginal conditions denoted by Equations 4.3 to 4.6. The theoretical model, using a generalized function form, suggests that the second order of the polynomial cost function will be particular important in determining the change in search intensity in response to a change in benefit levels. The empirical model is particularly focused on analyzing this latter response. Therefore, the quadratic cost function used in the present study is consistent with its broad use in the literature, the relationship implied by the theoretical model, and computational considerations.

In which X_i is a vector of individual characteristics, β is a vector of parameters and q_i represents the unobserved heterogeneity which is normally distributed with mean zero and variance 1.

The first order condition for the search intensity choice problem of Equation 4.2 using Equation 4.7 as the particular specification for the cost of search function yields:

$$\bar{s}_j = \frac{R_j(q)}{c_j} \tag{4.9}$$

with $R_j(q)$ as defined in Equations 4.4 and 4.6, $j = 1, 2, 3$. We added the argument q to express the dependence of R_j on the unobserved heterogeneity q, as q enters the calculation of the marginal returns R_j via market-determined part of job offer arrival rate λ. Note that Equation 4.9 defines the latent search intensity $\bar{s}_j$ that equate the marginal cost of search and the marginal returns to search.

To compute $\bar{s}_j$ in Equation 4.9, we need to calculate the marginal return to search $R_j(q)$ which by Equations 4.4 and 4.6 involves the computation of the expected income gain due to search $E(W(x) - V|x \geq \xi)$, represented by the integral $\int_{\xi}^{\infty}[W(x) - V]dF(x)$. In order to approximate $E(W(x) - V|x \geq \xi)$ with a value that we can evaluate, we need to use an assumption we mentioned in section 4.2: the present value of a future earning stream given a wage equal to x is $W(x) = x/\rho$, where $\rho > 0$ is the constant discount rate. Therefore, we approximate the expected income gain due to search by

$$\int_{\xi}^{\infty}[W(x) - V]dF(x) = \frac{1}{\rho}\int_{\xi}^{\infty}(x - \xi)dF(x) = \frac{1}{\rho}[E(x - \xi|x \geq \xi)] \tag{4.10}$$

As will be described in section 4.4, the weekly wages of those individuals who found a new job are observable in the data of Labor Market Activity Survey (LMAS). Hence, $\bar{s}_j$ and ξ are capable to be computed simultaneously using the available data.

Second, we provide a link between the observed search indicators s_j^o available in the data (in Equation 4.23 of section 4.4) and the (optimal) latent search intensities $\bar{s}_j$ that equate marginal cost of search and marginal return to search (in Equation 4.9). In the data, the transitions into work are observed for those who reported not to be searching. Reporting error or measurement error in the observed search indicators may be one reason for these observations [23]. For this reason, we incorporate reporting errors in the search indicators to capture the stochastics. In the data, there are two types of indicators: (1) dichotomous indicator, and (2) ordinal numbers. Obviously, both of them take discrete values. The choices characterized by the discrete data reflect the equilibrium choices for agents represented by the theoretical model. Let ϵ_1 denote the random error, and let s_1^o, denote the observed search indicator that reflects the basic thrift to search for a job. A dichotomous indicator variable s_1^o related to observed job search is defined by a probit model as:

$$
(4.11) \qquad \tilde{s}_1 = \bar{s}_1 + \epsilon_1 \;\; where \;\; \epsilon_1 \sim N(0,1), \;\; \tilde{s}_1^* = \max\{0, \tilde{s}_1\} \;\; and
$$
$$
s_1^o = \begin{cases} 1 & if \;\; \tilde{s}_1 > 0 \\ 0 & if \;\; \tilde{s}_1 \leq 0 \end{cases}
$$

In Equation 4.11, $\tilde{s}_1$ represents the stochastic equivalence of $\bar{s}_1$, the latent search intensity, since we assume that the observed search indicators measure the true search efforts with an error. Equivalently, let $\tilde{s}_1^*$ represent the optimal search intensity with a stochastic error which implies that $\tilde{s}_1^* = \max\{0, \tilde{s}_1\}$.

As the number of channels a searcher used, s_2^o, and the time she/he spent on searching since the last day worked, s_3^o, are variables that can only take ordinal numbers like $0, 1, 2, \ldots$, technically, it seems natural to model the count variables s_2^o and s_3^o by Poisson distribution. Hence, for individuals who are searching for a job, we specify:

$$
(4.12) \qquad \tilde{s}_j = \bar{s}_j + \epsilon_j \;\; where \;\; \epsilon_j \sim N(0,1), \;\; \tilde{s}_j^* = \max\{0, \tilde{s}_j\}, \; j = 2, 3 \;\; and
$$
$$
P(s_j^o = k | \tilde{s}_j > 0) = \frac{(\mu_j)^k \exp(-\mu_j)}{k!}, \;\; k = 0, 1, 2, 3, 4
$$

[23]Other reason is like the personal contact we mentioned in section 4.2.

where the mean parameters μ_j is defined by $\mu_j = \exp(\tilde{s}_j^*) > 0,\ j = 2,3$. It needs to be noticed that conditioning on $\tilde{s}_j > 0$ implies that equation 4.12 applies to searchers only. For individuals who report not to be searching, we assume that the Poisson distribution does not apply. For the non-searchers, the condition $\tilde{s}_j \leq 0$ holds which implies that $s_j^o = 0$. Note that in our specification, we allow for the observation of zero value of search channel or zero value of search time ($k = 0$) for individuals who report to search.

The stochastic errors of observed search indicators, $\epsilon_1, \epsilon_2,\ and\ \epsilon_3$ are assumed jointly normally distributed. Let $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3)'$, we specify:

$$(4.13) \qquad \epsilon \sim N(0, \Sigma_s) \quad with$$

$$(4.14) \qquad \Sigma_s = \begin{Bmatrix} 1 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & 1 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & 1 \end{Bmatrix} = \begin{Bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{Bmatrix}$$

where ρ_{ij} denote the correlation coefficient between ϵ_i and ϵ_j with $i = 1,2,3$ and $j = 1,2,3$. (Note the variance-covariance matrix is equivalent to the correlation matrix since $\sigma_{11} = \sigma_{22} = \sigma_{33} = 1$.) Since the search effort s is defined as a vector of three different search indicators $s = (s_1, s_2, s_3)$ in section 4.2, using the relation $\tilde{s} = \bar{s} + \epsilon$, Equation 4.13 implicitly defines the density function of $\tilde{s} = (\tilde{s_1}, \tilde{s_2}, \tilde{s_3})$, the stochastic equivalence of (optimal) latent search intensity, which is denoted by $g(\tilde{s}; \Sigma_s)$.

Last, we assume that the wage offers arrive from a log-normal distribution with the following form:

$$(4.15) \qquad \ln w = K'\gamma + u \quad where\ \ u \sim N(0, \sigma_u^2)$$

in which K is a vector of observable individual characteristics, γ is a vector of parameters and u is an error term that is normally distributed. However, the wages after transitions into work of a few respondents in the data are observed lower than their EI benefits. Such observations may be attributed to the measurement or reporting error. Other reasons may be explained as earning working experience valuable in the future or having social relationship which can not be measured by the value of money. Irrespective of the reason, we need to include a measurement error into the empirical specification of observed wage offers. The measurement error can be easily included (Wolpin, 1987) by adding a multiplicative measurement error v that is independent of the true wage w. By letting w^o denote the observed wages, we assume w^o are distributed as:

$$(4.16) \qquad \ln w^o = \ln w + v \quad \textit{where} \;\; v \sim N(0, \sigma_v^2)$$

In which the measurement error v, is supposed to be independent of u.

4.3.2 LIKELIHOOD FUNCTION

The parameters of the job offer arrival rate, the cost of search function, the wage offer distribution, and the parameters of the distribution of stochastic error in search indicators and measurement error in wages are estimated simultaneously by the method of simulated maximum likelihood. In this section, we will describe how the likelihood function for the model to be formulated.

Given an observation(t, s^o, w^o) consisting of duration of unemployment t, search indicators $s^o = (s_1^o, s_2^o, s_3^o)$ and the observed true wages after a transition from unemployment into employment w^o, first, we address the density function of duration t, conditional on the value of the (optimal) latent search intensity $\tilde{S}(q)$ (Equations 4.9, 4.11 and 4.12) and on the unobserved heterogeneity q. Letting transition rate (or hazard rate) from unemployment into employment be ϕ, we have the vector form of ϕ as:

$$(4.17) \qquad \phi = (\alpha_0 + \alpha\tilde{S}')\lambda[1 - F(\xi)] = \phi(\tilde{S}, \lambda, w) = \phi(\tilde{S}(q), q)$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and $\tilde{S} = (\tilde{s_1}, \tilde{s_1}\tilde{s_2}, \tilde{s_1}\tilde{s_3})$ as we defined in Section 4.2.

Therefore, the density function of unemployment duration given instantaneous hazard rate ϕ is:

$$(4.18) \qquad f(t|\phi) = \phi \exp\{-\phi t\}, \quad \forall\, t \in [0, \infty)$$

Obviously, ϕ is dependent on job offer arrival rate and on wage offer distributions. The dependence on arrival rate of job offers runs through search effort $\tilde{S}(q)$ (Equations 4.9, 4.11 and 4.12 imply that $\tilde{s_j},\ j = 1,2,3$ are functions of q) and unobserved heterogeneity q (enters via λ and $\tilde{s_j}$). The dependence on distribution of wage offers runs through $\bar{s}_j(q),\ j = 1,2,3$ (Equations 4.9, 4.4 and 4.6). Together with the facts that $\tilde{s_j}(q)$ are stochastic equivalences of $\bar{s}_j(q)$: $\tilde{s_j}(q) = \bar{s}_j(q) + \epsilon_j$ (Equations 4.11 and 4.12) and $\tilde{S}(q) = (\tilde{s_1}(q), \tilde{s_1}(q)\tilde{s_2}(q), \tilde{s_1}(q)\tilde{s_3}(q))$, transition rate from unemployment ϕ can be written as a function of $\tilde{S}(q)$ and q (as in Equation 4.17). Consequently, given $\phi(\tilde{S}(q), q)$, equation 4.18 yields the density function of spell of unemployment t, conditional on latent search intensity $\tilde{S}(q)$ and unobserved heterogeneity q as:

$$(4.19) \qquad f(t|\tilde{S}(q), q) = \phi(\tilde{S}(q), q) \exp\{-\phi(\tilde{S}(q), q)t\}, \quad \forall\, t \in [0, \infty)$$

The density function of $\tilde{s}(q) = (\tilde{s_1}(q), \tilde{s_2}(q), \tilde{s_3}(q))$ given q is defined by Equation 4.13 which will be denoted as $g(\tilde{s}(q); \Sigma_s)$ in the sequel.

Second, the observed wage offer distribution has already been discussed in Equation 4.16. However, the model implies that the distribution of the true wages for unemployed individuals is the truncated distribution to wage offers above the reservation wage ξ. Therefore, the density for true wages is $f(w)/[1 - F(\xi)]$, for $w > \xi$, and zero otherwise. We will let $g(w^o|w)$ represent the density function of observed wages w^o conditional on the true wages w below.

Third, the probability distributions of the observed search indicators s_1^o, representing search or not, s_2^o, representing the number of methods used for search and s_3^o, representing the time spent on search are defined by Equations 4.11 and 4.12, which will be denoted by $P(s_j^o|\tilde{s}_j(q))$, $j = 1, 2, 3$ in the following.

Finally, to complete the likelihood function, we need to multiply the various parts and integrate over the latent variables. For an unemployed worker who is searching for a job with a completed unemployment duration t, a vector of observed search indicators $s^o = (s_1^o, s_2^o, s_3^o)$ and an observed wage after a transition into employment w^o, we have the following likelihood function:

$$(4.20) \quad \int_q \int_{R(s^o)} f(t|\tilde{S}(q), q) P(s_1^o|\tilde{s}_1(q)) P(s_2^o|\tilde{s}_2(q)) P(s_3^o|\tilde{s}_3(q)) g(\tilde{s}(q); \Sigma_s) g(q) \int_\xi^\infty g(w^o|w) \frac{f(w)}{1 - F(\xi)} dw d\tilde{s}(q) dq$$

In Equation 4.20, the region of integration $R(s^o)$ is defined by the observed search indicators through Equations 4.11 and 4.12.[24] Note that the likelihood contributions for right-hand censored unemployment spells and for those who report not to search are straightforward simplification of Equation 4.20. More specifically, the likelihood contribution for a searcher who have not been observed a transition into work by the end of the survey period is:

$$(4.21) \quad \int_q \int_{R(s^o)} P(T \geq t|\tilde{S}(q), q) P(s_1^o|\tilde{s}_1(q)) P(s_2^o|\tilde{s}_2(q)) P(s_3^o|\tilde{s}_3(q)) g(\tilde{s}(q); \Sigma_s) g(q) d\tilde{s}(q) dq$$

where $P(T \geq t|\tilde{S}(q), q) = \exp[-\phi(\tilde{S}(q), q)t]$ denotes the survival function conditional on latent search intensity and unobserved heterogeneity. Additionally, the likelihood function of an individual who report not to be searching but is observed a transition into employment is represented by:

$$(4.22) \quad f(t|\phi) \int_\xi^\infty g(w^o|w) \frac{f(w)}{1 - F(\xi)} dw$$

[24]As $\tilde{S}(q) = (\tilde{s}_1(q), \tilde{s}_1(q)\tilde{s}_2(q), \tilde{s}_1(q)\tilde{s}_3(q))$, from Equations 4.11 and 4.12, $\tilde{s}_1(q) \in [0, 1]$, $\tilde{s}_1(q)\tilde{s}_2(q) \in [0, 4]$ and $\tilde{s}_1(q)\tilde{s}_3(q) \in [0, 4]$.

in which $\phi = \alpha_0 \lambda[1 - F(\xi)]$ is the exit rate from unemployment for non-searchers. Finally, for an individual who do not search and then have not got a job by the end of the survey period, the likelihood contribution is $P(T \geq t | s_1^o = 0, s_2^o = 0, s_3^o = 0) = \exp(-\phi t)$.

In the process of calculating the likelihood function, the problem of multidimensional integration of normally distributed random variables can be handled by the smooth recursive conditioning algorithm (SRC) for simulating multidimensional integrals over normally distributed random variables and applying simulated maximum likelihood (SML) (see Börsch-Supan and Hajivassiliou (1993)). The Monte Carlo integration involves the generation of random numbers for unobserved heterogeneity q and stochastic errors for the vector of latent search indicators $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$.

4.4 DATA

The Labour Market Activity Survey (LMAS) collected information on the annual labour market participation of Canadians and the characteristics of up to five job held in each of the calendar years from 1986 to 1990. The surveyed population is consist of all civilian, non-institutionalized persons aged 16-69, who were residents of the ten provinces in January 1987. Two longitudinal files (1986-1987, and 1988-1989-1990) provide information on the labour market participation and job characteristics of all responding individuals over two or three year period. This data set provides us with a good opportunity to track the working history, job search information and the transition into a new job between calendar years for each individual who reported to experience unemployment. We choose the second longitudinal file (1988-1990) in the LMAS and focus on the respondents who worked in the first year of survey (1988) and then experienced at least one spell of unemployment over the three-year long survey period. [25] Hence, we can observe both the transition into and out of from unemployment for respondents who found the new job by the end of the last year of survey (1990). Our sample have 1122 individuals among whom 1095 reported to be searching in their durations absent from work.

4.4.1 BACKGROUND VARIABLES AND SAMPLE STATISTICS

Survey respondents were asked to report their labor market states for 53 weeks of each calendar year in the survey period. This information can be used to construct the previous working spell and the duration of unemployment afterwards for respondents who were unemployed. The wage rates on all jobs are also observed.

Table 4.1 shows the sample statistics of weekly wage rates on the previous job and on the new job, unemployment duration and age for unemployed individuals who reported to search for a job and for those who did not. Mean wage at previous job of the searchers is lower than that of those who did not search. But after searching, the unemployed workers transited into new jobs with a higher rate of mean wage whereas the other unemployed who did not search worked at a lower mean wage after the transition into employment. The searchers have one week longer mean unemployment duration than those who did not search that may because the sample size of non-searchers is very small. Another reason for this observation may be the characteristics of the occupation of the unemployed who did not search.[26] In addition, the mean age of the searchers is lower than that of the non-searchers.

[25] For the respondents who reported to have more than one spell of unemployment after they previously worked in 1988, we only use the related information concerning their first spell of unemployment.

[26] From Table 4.2, most unemployed workers who did not search are in the occupation 4 (Fishing, Mining, Manufacture, Construction and Transportation). For them, seasonal layoff may be the reason absent from work.

Table 4.1: Sample Statistics of Weekly Wage Rates and Unemployment Duration

Variable	Searchers (n=1095)		Non-searchers (n=27)	
	Mean	Standard Deviation	Mean	Standard Deviation
Weekly Wage at Previous Job	338.11	222.57	362.83	206.34
Weekly Wage at New Job	372.54	256.08	355.72	218.23
Unemployment Duration	16.98	14.55	15.85	12.04
Natural Log of Age Group	1.288	0.42	1.35	0.31

As we assumed that the exogenous part of the job offer arrival rate, λ, and the true wage rates, w, are determined by the individual characteristics, we need to specify the related background variables. The available background characteristics are age, level of education, sector of occupation and region of residence. We define three education dummies for the level of education, educ1: 0-8 years or some secondary education, educ2: post-secondary cert. or diploma, educ3: university degree or trade certificate or diploma (educ4: graduated from high school serves as reference group), four occupation dummies for sector of occupation, occu1: administration managerial; religion, teaching, medical or artist occupation, occu2: science and engineers, occu3: clerical or sales occupation, occu4: farming, mining, manufacture, construction and transportation occupation (occu5: service occupation serves as reference occupation) and three area dummies for the region of residence, area1: the strongly industrialized central region of Canada, area2: the prairie provinces which is known for its agricultural industry and energy resources, area3: the west coast of Canada which is characterized by tourist industry and natural resources (area4: the Atlantic region which is known for the fishing industry serves as the reference region).[27]

Table 4.2 provides sample statistics of background dummy variables. The unemployed workers (search or not) concentrate at the less professional sector of occupation (occu4), living in the less industrialized region (area4) with lower level of education (educ1).

[27]Area1 includes Quebec and Ontario, area2 includes Manitoba, Saskatchewan and Alberta, area3 includes British Columbia and area4 includes Newfoundland, Prince Edward Island, Nova Scotia and New Brunswick.

Table 4.2: Sample Statistics for Background Variables

Variable	Searchers (n=1095)		Non-searchers (n=27)		Total (n=1122)	
	Sample Size	Sample Percent	Sample Size	Sample Percent	Sample Size	Sample Percent
Educ1	442	40.37	18	67	460	41
Educ2	253	23.11	0	0	253	22.55
Educ3	140	12.79	1	3.7	141	12.57
Educ4	260	23.74	8	29.63	268	23.89
Area1	331	30.23	9	33.3	340	30.3
Area2	213	19.45	2	7.41	215	19.16
Area3	99	9.04	0	0	99	8.82
Area4	452	41.28	16	59.26	468	41.71
Occu1	103	9.41	2	7.41	105	9.36
Occu2	30	2.74	2	7.41	32	2.85
Occu3	257	23.47	3	11.11	260	23.17
Occu4	532	48.58	19	70.37	551	49.11
Occu5	173	15.80	1	3.7	174	15.51

4.4.2 INDICATORS OF SEARCH INTENSITY

Detailed information on search behavior of unemployed respondents was collected in each year of survey. The related sample questionnaire will be shown in the Appendix C.1.

Using the information available in the data, we can construct two measures on job search behavior; one for "search or not" and two for "search intensity":

(a) Search : a dummy variable indicating whether or not the unemployed person did search a job during the absence from work.

(b) Search intensity index 1 (channel index): a measure of search intensity taking on values $0, 1, 2...4$, based on how many search channels the respondent used during the spell of unemployment to look for a job.

(c) Search intensity index 2 (time index): a measure of search intensity taking on values $0, 1, 2...4$, based on the number of weeks the unemployed individual spent on job search since the last day of work.

The construction of these two measurements on job search behavior will be described in Appendix C.2. More specifically, we define three observed job search indicators for unemployed individuals as following:

$$(4.23) \quad s_1^o = \begin{cases} 1 & if\ did\ search\ a\ job\ when\ absent\ from\ work \\ 0 & if\ did\ not \end{cases}$$

$$s_2^o = 0, 1...4 \quad number\ of\ search\ channels\ used\ by\ a\ job\ seeker$$

$$s_3^o = 0, 1...4 \quad the\ proportion\ of\ time\ spent\ on\ searching\ by\ a\ job\ seeker\ to\ the\ whole\ joblessness\ duration$$

Table 4.3 contains the sample statistics of observed indicators of search. In our sample, for the unemployed who reported to be looking for a job ($s_1^o = 1$), we do observe positive values on the number of channels used for searching ($s_2^o > 0$) and the amount of time spent on search ($s_3^o > 0$) in the meanwhile. Comparatively, for the unemployed who were not searching ($s_1^o = 0$), we observe zero values of both s_2^o and s_3^o ($s_2^o = s_3^o = 0$).

Table 4.3: Sample Statistics of Observed Search Indicators

Variable	Searchers (n=1095)		Non-searchers (n=27)	
	Mean	Standard Deviation	Mean	Standard Deviation
s_1^o	1	0	0	0
s_2^o	2.55	1.07	0	0
s_3^o	2.71	1.17	0	0

4.4.3 UNEMPLOYMENT INSURANCE

The LMAS does not provide observations on the value of unemployment insurance (EI) [28] and the maximum entitlement to EI benefits for individuals who report to be unemployed. But we can calculate them using the related information available in the data. Eligibility for unemployment insurance in Canada depends on both an individual's work history and the unemployment rate in his/her local labor market (Unemployment Insurance Act, Archived). The LMAS records the employment spell of last job for those who are not currently employed but worked in the previous year. For this reason, we construct a variable ELIG by comparing

[28] The unemployment insurance is called employment insurance in Canada. As a result, we represent unemployment insurance as EI in the sequel.

an currently unemployed individual's working history with the entrance requirement for EI applicable in his/her economic region:

$$(4.24) \qquad ELIG = \begin{cases} 1 & if\ the\ individual\ is\ eligible\ to\ EI \\ 0 & if\ the\ individual\ is\ not\ eligible\ to\ EI \end{cases}$$

The rate of weekly benefits payable to an eligible EI claimant is an amount calculated according to the Canadian EI legislation as follows, to a maximum of 55 percent of the maximum weekly insurable earning (MWIE)[29]:

$$(4.25) \qquad weekly\ benefits = 55\% \times AWIE$$

in a general case. Where AWIE represents the average weekly insurable earnings. In any case that the claimant or their spouse supports one or more persons who are dependants of the claimant or of their spouse;

$$(4.26) weekly\ benefits = \begin{cases} 60\% \times AWIE & if\ AWIE \leq\ 50\% \times MWIE \\ \max\{55\% \times AWIE,\ 30\% \times MWIE\} & if\ AWIE >\ 50\% \times MWIE \end{cases}$$

As LMAS records the wages at previous jobs for those who currently unemployed, we can easily calculate the weekly rate of EI benefits by the rules defined above. In addition, the maximum entitlement to the EI benefits also depends on the number of weeks of previous insurable employment and the regional rate of unemployment. Similarly, it can be computed using the data available.

Table 4.4 provides with the sample statistics of value of weekly EI benefits and maximum entitlement to EI for the unemployed who were searching and for those who were not. Both the mean weekly EI benefits and the mean maximum EI entitlement of the non-searchers are higher than those of the other unemployed who were searching for a job.

[29]The maximum weekly insurable earning of an insured person is an amount calculated by multiplying one hundred and eighty-five dollars by the Earning Index for the year (Unemployment Insurance Act, Archived).

Table 4.4: Sample Statistics of EI Benefits and Maximum Entitlement to EI

Variable	Searchers (n=1095)		Non-searchers (n=27)	
	Mean	Standard Deviation	Mean	Standard Deviation
Weekly EI Benefits	147.27	124.85	198.98	108.10
Maximum Entitlement to EI	28.19	16.96	32.07	7.10

4.5 EMPIRICAL RESULTS

The estimation results of empirical model are presented in this section. Parameter estimates are shown in Table 4.5 to Table 4.8. Group averages of the optimal values of reservation wage and search intensity for each group of unemployed workers who are classified by personal and regional characteristics are reported in Table 4.9.

4.5.1 PARAMETER ESTIMATES

We use 100 replications for the error distribution of search indicators to simulate the integrals in Equations 4.20 and 4.21 using the simulated maximum likelihood method [30]. The rate of discount ρ is assumed to be fixed at 5%. Additionally, the fixed cost of search c_0 is assumed to be equal to zero for simplicity.

[30] The observed search indicators (Table 4.3) show that $s_2^o > 0$ and $s_3^o > 0$ when $s_1^o = 1$ and $s_2^o = s_3^o = 0$ when $s_1^o = 0$. Therefore, $s = (s_1, s_2, s_3)$ should be a good approximation of $S = (s_1, s_1 s_2, s_1 s_3)$. Consistent with the data, we use $\tilde{s}$ instead of $\tilde{S}$ to calculate the exit rate from unemployment in the density function of unemployment spell when simulate the integrals in Equations 4.20 and 4.21.

Table 4.5: Parameter Estimates for the Job Offer Arrival Rate

Market-determined Part of Job Offer Arrival Rate λ			
Variable	Coefficient	Standard Error	P-value
Ln(age group)	0.71506**	0.10707	2.41e-11
Square of ln(age group)	-2.1198**	0.090611	0
Educ1	-0.17815**	0.010184	0
Educ2	0.11098**	0.0049767	0
Educ3	0.25198**	0.0197	0
Area1	0.20561**	0.0036153	0
Area2	-0.09657**	0.0017281	0
Area3	0.10904**	0.0037229	0
Occu1	-0.62258**	0.0093147	0
Occu2	0.053418*	0.0017387	0
Occu3	-1.7855**	0.096632	0
Occu4	-1.2173**	0.34568	0.00042896
Search Effective Parameter α			
Variable	Coefficient	Standard Error	P-value
Intercept α_0	4.1401**	0.72942	1.38e-08
Search or not α_1	0.4572**	0.021574	0
Search channel α_2	0.8085**	0.076021	0
Search time α_3	0.79527**	0.084602	0

** represents significant at 5 percent level.

Table 4.5 provides the parameter estimates for the job offer arrival rates. The market-determined part of the job offer arrival rate, λ, decreases with age for the unemployed individuals who exceed the age of 24. The highly educated individuals have more opportunities to receive job offers (λ increases with the level of education). The prairie provinces (area2) have the lowest job offer arrival rate. Whereas in the more industrialized area (area 1 and area3), the unemployed individuals have higher arrival rates. Moreover, the engineers (occu2) have the most market opportunities while the clerk and the salesman (occu3) who just followed a general type of education have the lowest value of arrival rate. The coefficient estimates of effectiveness of search indicators are displayed in the lower part of Table 4.5. As predicted by the theoretical model, all coefficients are positive. Clearly, three search indicators all influence the job offer arrival rate significantly. The number of channels used for searching has the largest effect on the arrival rate compared with other search indicators.

Table 4.6: Parameter Estimates for the Error Distribution of Search Indicators

Variable	Coefficient	Standard Error	P-value
$\sigma_{12} = \rho_{12}$	0.39147**	0.023394	0
$\sigma_{13} = \rho_{13}$	0.30455**	0.011339	0
$\sigma_{23} = \rho_{23}$	0.26917**	0.0031343	0

Note: ρ_{12}, ρ_{13} and ρ_{23} are the correlation coefficients between s_1 (indicating search or not) and s_2 (indicating number of search channels used), s_1 and s_3 (indicating the amount of time spent on search) and s_2 and s_3 respectively.

Table 4.7: Parameter Estimates for the Cost of Search Function

Variable	Coefficient	Standard Error	P-value
c_1	25.573**	0.75351	0
c_2	25.831**	2.2538	0
c_3	24.12**	1.5088	0

Table 4.6 contains the estimates of the correlation coefficient of the reporting errors of the search indicators. All estimates of the correlation coefficients are positive and significant. Note that the estimated correlation (covariance) matrix is positive definite. Table 4.7 shows the parameter estimates of the cost of search function. All estimates are significant and search channel indicator s_2 has the highest marginal cost of search.

Table 4.8: Parameter Estimates for the Wage Offer Distribution

Variable	Coefficient	Standard Error	P-value
Constant	2.7481**	0.48229	1.21e-08
Ln(age group)	1.2559**	0.12961	0
Square of ln(age group)	-0.16062**	0.021989	2.78e-13
Educ1	-0.13099**	0.019456	1.66e-11
Educ2	0.081665**	0.0048246	0
Educ3	0.29**	0.013341	0
Area1	0.22085**	0.0090998	0
Area2	0.14221**	0.0032724	0
Area3	0.20762**	0.0031078	0
Occu1	0.2746**	0.02853	0
Occu2	0.2838**	0.016514	0
Occu3	0.091734**	0.0054318	0
Occu4	0.60142**	0.047241	0
σ_u	0.92338**	0.17994	2.87e-07
σ_v	0.2627**	0.08987	0.0034662

Note: σ_u is the standard deviation of wage offer distribution and σ_v is the standard deviation of the measurement error of observed wages.

Table 4.8 provides the report of parameter estimates of the wage offer distribution. The wage offer increases with age when the unemployed are young and decreases with age when they are older than 54. Individuals with the highest education level (educ3) obtain the highest wage offers. The agricultural area (area2) provides the lower value of wages and the more industrialized area (area1 and area3) provides the higher level of wages. The workers in the construction and transportation occupations (occu4) achieve the highest wage offers while the clerks (occu3) receive the lowest value of wage offers. The standard deviation of the wage offer distribution, σ_u, is much greater than the standard deviation of the reporting error in observed wages, σ_v, which implies that the most part of the variation in observed wages is attributed to the variation in wage offers.

4.5.2 GROUP RESULTS FOR ENDOGENOUS VARIABLES AND LABOR MARKET TRANSITION RATES

We obtain the optimal values of reservation wage, ξ, and search effort, $\bar{s} = (\bar{s_1}, \bar{s_2}, \bar{s_3})$, by substituting the parameters of estimation into the structural model. The exit rate from unemployment, ϕ, is then calculated using the resulting values of ξ and $\bar{s}$. The group average results for people who are differentiated in personal or regional characteristics such as level of education and region of residence are shown in Table 4.9.

Table 4.9: Group Average Values for Reservation Wage, Search Intensity and Transition Rate into Employment

Group	Group Size	Group Average Values				
		ξ	$\overline{s_1}$	$\overline{s_2}$	$\overline{s_3}$	ϕ
Educ1	460	233.68	0.31695	0.5549	0.58453	0.049125
Educ2	253	299.18	0.48119	0.84242	0.88742	0.10471
Educ3	141	394.8	0.54664	0.95701	1.0081	0.047771
Educ4	268	270.85	0.42793	0.74918	0.7892	0.063223
Area1	340	296.53	0.42569	0.74526	0.78507	0.07742
Area2	215	285.93	0.40261	0.70485	0.74249	0.064353
Area3	99	305.03	0.41084	0.71926	0.75768	0.060779
Area4	468	254.16	0.40028	0.70078	0.73821	0.056825
Occu1	105	387.15	0.55084	0.96437	1.0159	0.054523
Occu2	32	335.1	0.54968	0.96232	1.0137	0.081761
Occu3	260	212.16	0.29707	0.52008	0.54785	0.040593
Occu4	551	304.22	0.42927	0.75152	0.79166	0.059911
Occu5	174	214.25	0.40293	0.70541	0.74309	0.11991

First of all, Table 4.9 shows that both the reservation wage and the search intensity increase with the level of education. Individuals with the highest education (educ3) search the most but have a low transition rate into employment. It is attributed to the factor that they are highly demanding on the reservation wage. Comparatively, the unemployed with the post-secondary diploma (educ2) obtain the highest rate of transition due to their plenty of search effort and relatively lower level of reservation wage. While the workers with the lowest education level (educ1) tend to be the ones the most inactive on job search. And they end up with a low transition rate into employment. Besides the factor of search effort, the limited opportunities for the lowly educated person on the job market may be another reason of low transition rate for these people. Secondly, the person live in the more industrialized region (area1 and area3) search more and ask for higher reservation wage. The unemployed in Ontario and Quebec (area1) region search most intensively and achieve the highest transition rate into employment. Whereas the unemployed in the Atlantic region (area4) search less and exit from unemployment with a slower speed compared with the others. Finally, the more professional the industry is, the more intensive the workers search and the more reservation wage they ask for. The managers (occu1) and engineers (occu2) are those with the most sufficient search effort and the greatest value of reservation wage. The latter (occu2) obtains the second-highest exit rate from unemployment. In contrast, the individuals who are lack of skill specific training, for instance, the clerks and salesmen (occu3) search the least and transit into work with a very low speed even though the level of reservation wage they request is not high. While the workers in the service industry (occu5) achieve the highest transition rate which is associated with their sufficiently low reservation wage and the relatively proper search effort. In addition, the special characteristics of service industry (such as the seasonal layoff and recruitment and the plenty of temporary job opportunities) could be another explanation for the high transition rate.

The estimation results for endogenous variables show that the workers who are highly educated, more professional and living in the industrialized area search more and demand higher reservation wages. As we discussed in last subsection, results of Table 4.5 show that the market-determined part of job offer arrival rate which depends on the characteristics of individuals is also higher for these people. Consequently, the reports of empirical estimation suggest that the unemployed individuals who are facing more opportunities on the market search more intensively due to their higher value of return to search which is consistent with the result predicted by the theoretical model. Moreover, the search effort does influence the job finding success effectively. The numbers in Table 4.9 suggest that the unemployed with a higher level of search effort and a relatively lower value in reservation wage transit into work sooner compared with the others.

4.6 SIMULATION OF POLICY CHANGES ON THE UNEMPLOYMENT COMPENSATION SYSTEM

Unemployment insurance (EI) programs aim at insuring unemployed workers against loss of income but are often associated with the adverse incentive effects. Generous unemployment benefits seem to decelerate the transition into work by lowering the search intensity and increasing reservation wages. To see the impacts of unemployment compensation system on the exit rate from unemployment, let's first take a look at Table 4.10 which contains the group average values for the weekly EI benefits, b, maximum EI entitlement, T, and exit rate from unemployment, ϕ, for the unemployed individuals classified by the eduction level and occupation sector.

Table 4.10: Average Values of EI Benefits and Transition Rate into Employment for Specific Groups of People

Group	Group Size	Group Average Values		
		Weekly EI Benefits (b)	Maximum EI Entitlement (T)	Transition Rate (ϕ)
Educ1	460	139.03	26.513	0.049125
Educ2	253	153.44	29.174	0.10471
Educ3	141	217.81	33.106	0.047771
Educ4	268	138.97	27.963	0.063223
Occu1	105	204.41	32.79	0.054523
Occu2	32	161.16	30.781	0.081761
Occu3	260	126.17	30.396	0.040593
Occu4	551	171.77	27.347	0.059911
Occu5	174	95.733	24.943	0.11991

Table 4.10 shows that the highly educated workers (educ2 and educ3) are those who receive a greater value of insurance benefits in a longer duration eligible to EI. Apparently, with a high reservation wage demand, the University graduates (educ3) do not transit into work soon. Comparatively, the unemployed with lower education levels (educ2 educ4 and educ1) who receive both the less amount in weekly benefits and the shorter entitlement to EI are less demanding on reservation wages. Therefore, they exit from unemployment earlier than the University Graduates. The workers in occupation 1 (managers, doctors, teachers and etc.) obtain the highest level of weekly benefits and maximum EI entitlement. They do not

get back to work soon because they are highly demanding on the reservation wages. Whereas the engineers (occu2) who get a lower value in EI in a shorter duration transit into work with a much higher speed. The unemployed clerks (occu3) receive the EI benefits for a relatively longer spell even though the value of weekly compensation is not that high. Hence, they do not have much incentive to search for a job and, therefore, they exit from unemployment with a lowest rate. Comparatively, the workers in occupation 4 (constructors, drivers and etc.) obtain insurance benefits in a relatively shorter duration. As a result, they search more intensively and find a job sooner than the clerks. The numbers in Table 4.10 suggest that unemployment insurance system do have effects on the choice of search effort and reservation wages for the unemployed individuals. The more generous the compensation, the lower the search intensity and the higher the reservation wages. Permanent benefit cuts or monitoring and benefit sanctions for insufficient search may serve as possible ways to restore incentives for unemployed workers to search for a job and reduce demand on reservation wages in the meanwhile. There is a need to evaluate the impact of different policy changes. Once the structural parameters of the model have been estimated, we can simulate the effects, on the search intensity, reservation wage and transition rate into employment, of a reform of the unemployment compensation system.[31] Two alternative policy changes are examined: (1) a 10% cut in the amount of unemployment insurance (EI) benefits for all who are unemployed; (2) imposition of a punitive sanction which incurs a 20% deduction in the EI benefits for those who do not search sufficiently. The two reforms on the compensation system produce the equivalent value of benefit save for the government. Comparison of the simulation results will thus allow us to assess the effects of these two reforms on the search behavior and labor

[31]The coefficients of the structural model are assumed to be constant as it is also assumed as Abbring, van den Berg and van Ours (2005). Specifically, the individual characteristics are assumed to remain unchanged over the spell of unemployment. In addition, the parameters in the cost of search function and the variances of the stochastic errors in the wage offer distribution and in the observed search indicators are also assumed to be time-invariant. Furthermore, 10 to 20 percent changes in the mean insurance benefits are relatively not large incremental changes given the data set.

market transition rate for the unemployed workers. The simulation results are reported in Table 4.11. The comparison of the consequences of two alternative reforms are recorded in Table 4.12.

4.6.1 A PERMANENT CUT IN INSURANCE BENEFITS

The simulation result of reform 1 for specific groups of people who are differentiated in education level and sector of occupation is reported in the middle part of Table 4.11 (the group results before reforms are recorded in the top of the Table 4.11 for use of comparison). The percentage changes of the search intensity, reservation wage and exit rate from unemployment with the benefits cut is recorded in Table 4.12. Corresponding to the ten percent permanent cut in insurance benefits, the unemployed with the highest education (educ3) who tended to be most active but highly demanded in the meantime reduce their reservation wage properly and rise their search intensity mostly, therefore, they achieve the greatest growth rate in transition rate. Similarly, the high school graduates (educ4) also become less demanding and search more intensively after the benefit cut to improve their transition rate sufficiently. In contrast, the workers with the lowest eduction level (educ1) who were the most less likely to search decrease their demand on reservation wage most and arise their search effort accordingly. However, the transition rate into employment is not improved much. That may be attributed to the reason that the lowly educated person face much less opportunities on the job market. The second-highly educated workers (educ2) were those less active than the University graduates but also less demanding on the reservation wages. With the benefits cut, the rise in their search intensity is great (second-high speed) but the decrease in their reservation wage is small and the resulting increment in transition rate is not large. However, the workers with education 2 still achieve the highest exit rate after the reform of the insurance compensation given a high exit rate from unemployment before the policy changes. It implies that the person with post-secondary education are most easily to transit into work after searching compared with the others.

The professional workers in occupation 1 raise their search intensity with the greatest speed and decrease their reservation wages moderately responding to the decline in EI benefits to obtain the greatest growth in transition rate. Workers in occupation 3 who did not search intensively before reduce the level of reservation wages mostly and increase their search effort with a high speed (second-high speed) at the same time with the cut in EI. As a result, the rate of increase in their exit rate from unemployment is secondly high. Similarly, the blue-collar workers in construction and transportation industries (occu4) also cut their reservation wages sharply and become more intensively on job searching with the deduction in benefits. Hence, the growth in their transition rate is large as well. The engineers (occu2) do not lower the value of reservation wage much but they do rise their search effort moderately when the decrease in EI insurance takes place. They thus also obtain a sufficient increment in the exit rate from unemployment which implies that it is easy for them to get back to work soon after search. While the workers in service industry (occu5) who had the highest exit rate from unemployment before the reform on the compensation system do not search much more and do not cut demand on reservation wage heavily as well when the weekly insurance benefits decline. Hence, their growth rate in transition rate is very small. It seems that the benefit cut do not influence the optimal choices of the unemployed who work in the service industry very effectively. It may be explained by the following factors: First, compared with the others, the value of EI benefits for the service industry workers is already very low which implies that the further deduct in insurance compensation could not have much impacts on the search behavior of the unemployed workers (For instance, the reservation wage do not have much space to be lowered). Second, the service industry provides a plenty of temporary job opportunities such as seasonal job openings for the employees. Therefore, the unemployed workers in service industry still could transit into work with a high rate after the benefit cut is implemented even though they did not rise search effort very much accordingly (Table 4.11 shows that the transition rate after reform 1 for the workers in the occupation 5 is still the highest). However, the payment and benefits for their jobs are not good enough. Table 4.10 shows that the value of weekly EI benefits and maximum entitlement to EI of workers in occupation 5 is the lowest. Third, the unemployed in the service industry are those most easily to end up with a job with low payment and low benefits which implies that the options they are facing on the market are also limited.

The simulation result of reform 1 reveals that when the benefits cut is implemented, the highly educated workers in a more professional industry would like to raise search effort much more but less likely to lower the reservation wages while the low educated workers in a less professional industry would prefer to reduce the reservation wage very sharply and arise search intensity moderately in order to get back to work soon. It is associated with the fact that the more educated person will have more chances to receive better job offers after searching on the job market.

4.6.2 MONITORING AND BENEFITS SANCTION FOR INSUFFICIENT SEARCH

In reform 2, we suppose the search behavior of the unemployed could be observed and monitored. A benefit sanction of 20 percent cut in weekly EI compensation would be imposed to those who did not provide proofs of active search ($s_2^o < 2$ or $s_3^o <= 2$). The save on the benefits from reform 2 is equivalent to that received from reform 1. Therefore, we could compare the impacts of the two policy changes of the compensation system from which the government could obtain the same amount of benefits save on the search behavior of the unemployed workers. The simulation result is shown in the last part of Table 4.11

and the percentage changes on the related variables are reported in Table 4.12. Clearly, the percentage of the people who got punished is higher for the workers with education 1 and education 2 which implies that there are more people in these two groups who did not search sufficiently before the reform. Similarly, the workers in occupation 3 and occupation 4 also searched less intensively before the benefit sanction is imposed. The highly educated person with University degree (educ3) who searched most intensively but also asked for the most amount of reservation wage reduce their demand on reservation wage moderately and increase their search effort properly in response to the benefit sanction. Additionally, they get the greatest growth rate in transition rate into employment. The high school graduates become much less demanding on the reservation wage and search more intensively than before. Hence, their increment in transition rate is high as well. The workers got post-secondary education (educ2) who were less active before rise their search effort in the highest speed after the benefits sanction takes place but do not cut their demand on reservation wage heavily. The improvement on their transition rate is also satisfactory here which implies that the workers with the post-secondary diploma could get back to work soon after search due to the sufficient opportunities they are facing on the market and the proper value of reservation wage they request. In contrast, the low educated individuals (educ1) who were also less likely to search for a job reduce the value of reservation wage mostly and in the meanwhile they do highly increase their effort on search corresponding to a sanction in benefits. However, the improvement in their exit rate from unemployment is limited. Thus, the benefit sanction for insufficient search does not improve the situation for the low educated person as much as it does for the others. As we discussed in reform 1, it may be that the less educated person do not have sufficient job opportunities on the labor market.

The engineers (occu2) who used to search actively arise their search effort moderately and cut their demand on reservation wage slightly with the benefit sanction. Yet, they still obtain great achievement on exit rate from unemployment (second-high speed). It is clearly show that the scientists and engineers (occu2) are those who are the most easily to transit into work after search on the market. Comparatively, another group of searchers, the managers and doctors in occupation 1, who searched actively but demanded more on the reservation wages also reduce their reservation wage a little but rise their search effort much more when the sanction is implemented. However, the growth in transition rate they achieve is not as high as the engineers do. That may be associated with the higher reservation wage they demand. The clerks and salesman (occu3) are those who were the most inactive in search before the reform. However, after the benefit sanction, they reduce their value of reservation wage and rise their search intensity both at the most. As a result, they obtain the greatest increment in transition rate. Similarly, the previous less active workers in construction or transportation industries (occu4) also become much more less demanding and search more intensively after the imposition of the sanction. Therefore, the growth in their transition rate is also great. Finally, the inactive searchers in the service industry reduce demand on reservation wage moderately and raise search effort accordingly. Conversely, the transition rate into work is not improved sufficiently. As we discussed in last subsection, it is also due to the limited job offers they receive on the market.

4.6.3 COMPARISON OF THE CONSEQUENCE OF THE TWO ALTERATIVE POLICY CHANGES

With the imposition of benefit cut and benefit sanction, the unemployed workers do raise search effort and reduce reservation wage at the same time to improve labor market transition rate which is consistent with the prediction of the theoretical framework. The decreases in average value of reservation wage are lower with the benefit sanction compared with that with the permanent benefit cut for all groups of people except the workers in the service industry. The declines for workers with education 2, education 3 in the occupation 1 and occupation 2 are especially small. It is associated with the fact that the remaining workers who did not get punished in these groups keep asking for reservation wage at previous levels. Together with the fact that the number of people in each group who got benefit sanction is not so many (less than one half), the reduces in group average value of reservation wage for these people are less. In contrast, the workers in the service industry get the lowest level of benefits and therefore the value of reservation wage they demand is already small. After about 40 percent of workers got punished by 20 percent deduct in EI benefits for insufficient search, the group average value of reservation wage is dragged very low. Therefore, to the unemployed who work in the service industry, the benefit sanction is a more strict way of policy change.

Compared with permanent benefit cut, the benefit sanction influences the search behavior more effectively for those who were less likely to search before the reforms take place. The workers with education 1 and education 2 who tended to be less active search more intensively with the benefit sanction and therefore obtain the greater growth in their transition rate into employment. Similarly, the less active searchers who work in occupation 3, occupation 4 and occupation 5 also increase their search effort more with the benefit sanction. Correspondingly, the increment in their transition rate is larger with benefit sanction. In contrast, the workers who used to search intensively do not rise search effort with benefit sanction as much as they do with benefit cut since the punishment for insufficient search does not apply to active searchers. The group average values of search intensity for these people are thus lower with benefit sanction. For instance, the workers with educ3 and educ4 who tended to search actively raise their search effort less with benefit sanction. However, they do achieve the higher improvement in their transition rate compared with they do with benefit cut. It is attributed to the reason that the university and high school graduates are potentially easily to transit into work after search and deduction in reservation wages. Moreover, the 20 percent deduct in weekly EI benefits is a more harsh punishment for the unemployed who did not search intensively in these groups. When the previous inactive ones got punished, they would highly increase their search effort and become less demanding on the reservation wages than they would with benefit cut. Therefore, the exit rate from unemployment is raised by them much more and the average value of the transition rate for the whole group is thus driven up even though the remaining searchers in the group do not rise search effort sufficiently. The same explanation applies to the active workers in occupation 2 (engineers) who also search less but obtain the greater increment in transition rate with benefit sanction. Whereas another group of searchers, the workers in occupation 1 (managers, doctors and etc.), who used to be the most active but search not as intensively as they do with benefit cut, do not make a better progress in their transition rate with benefit sanction. It may be explained as follows: the workers in occupation 1 have the highest demand on reservation wages and the number of people in this group who got punished for insufficient search is relatively small. Therefore, for them, the group average value of reservation wage with benefit sanction is higher than that with benefit cut. The managers could not get much improvement in their transition rate into work since their strong highly demand on reservation wages can not be reduced sufficiently with benefit sanction. Consequently, the benefit sanction seems to be less effective to the workers in occupation 1. However, the benefit sanction does have more obvious impacts on the search behavior of the workers who did not search actively and thus rise their transition rate much higher than the benefit cut does. In addition, previous searchers in other groups also obtain better improvement in their transition rate with benefit sanction compared with they do with benefit cut. As a result, when the number of workers in occupation 1 among the unemployed is not very large, the monitoring and benefit sanction for insufficient search may serve as a more effective way to stimulate unemployed workers to get back to work.

However, it should be noted that the deduction in insurance benefits could not improve the transition rate effectively for some specific groups of people. The lowly educated workers (educ1) and the workers in the service industry (occu5) can not make sufficient progress in their transition rate into employment even though they increase their search effort with a low reservation wage. It is due to the insufficient market opportunities they are facing. To these people, lowering benefits may be a less effective policy tool. The government may consider to offer more programs of education and training courses for specific skills to encourage the low educated person to improve themselves and get back to work soon.

Table 4.11: Simulation Results for the Two Alternative Policy Changes on Insurance Compensation System

Variable	Group Average Before Reforms								
	educ1	educ2	educ3	educ4	occu1	occu2	occu3	occu4	occu5
N	460	253	141	268	105	32	260	551	174
b	139.03	153.44	217.81	138.97	204.41	161.16	126.17	171.77	95.733
ξ	233.68	299.18	394.8	270.85	387.15	335.1	212.16	304.22	214.25
$\overline{s_1}$	0.31695	0.48119	0.54664	0.42793	0.55084	0.54968	0.29707	0.42927	0.40293
$\overline{s_2}$	0.5549	0.84242	0.95701	0.74918	0.96437	0.96232	0.52008	0.75152	0.70541
$\overline{s_3}$	0.58453	0.88742	1.0081	0.7892	1.0159	1.0137	0.54785	0.79166	0.74309
ϕ	0.04913	0.10471	0.04777	0.06322	0.05452	0.08176	0.04059	0.05991	0.11991
Variable	Group Average After Reform 1								
	educ1	educ2	educ3	educ4	occu1	occu2	occu3	occu4	occu5
N_1	460	253	141	268	105	32	260	551	174
$N_1/N(\%)$	100	100	100	100	100	100	100	100	100
b_1	125.13	138.1	196.03	125.07	183.97	145.05	113.55	154.59	86.159
ξ_1	225.94	293.69	386.68	265.18	380.94	330.98	205.65	296	210.78
$\overline{s_{11}}$	0.33595	0.5113	0.58494	0.45224	0.59023	0.58282	0.31695	0.45558	0.42253
$\overline{s_{21}}$	0.58816	0.89514	1.0241	0.79175	1.0333	1.0204	0.55489	0.79759	0.73973
$\overline{s_{31}}$	0.61957	0.94295	1.0788	0.83404	1.0885	1.0749	0.58453	0.84019	0.77924
ϕ_1	0.04981	0.10612	0.04899	0.06430	0.05557	0.08311	0.04131	0.06090	0.12133
Variable	Group Average After Reform 2								
	educ1	educ2	educ3	educ4	occu1	occu2	occu3	occu4	occu5
N_2	191	109	54	107	42	12	116	225	66
$N_2/N(\%)$	41.52	43.083	38.298	39.93	40	37.5	44.62	40.835	37.93
b_2	124.37	138.19	198.18	125.16	185.86	148.16	113.26	154.42	85.449
ξ_2	226.12	294.56	387.25	265.31	382.79	331.85	206	296.3	210.4
$\overline{s_{12}}$	0.33689	0.51281	0.5788	0.45199	0.58631	0.5736	0.31887	0.45554	0.42313
$\overline{s_{22}}$	0.5898	0.89778	1.0133	0.7913	1.0265	1.0042	0.55826	0.79752	0.74078
$\overline{s_{32}}$	0.6213	0.94573	1.0674	0.83356	1.0813	1.0578	0.58808	0.84012	0.78034
ϕ_2	0.04986	0.10654	0.04902	0.064606	0.05546	0.08357	0.04161	0.061077	0.1215

Note that N is the sample size for each group. N_i is the number of people and N_i/N is the percentage of people who got benefit deduction in reform i respectively where $i = 1, 2$.

Table 4.12: Comparison of the Impacts of the Two Alternative Reforms on the Search Behavior and Transition Rate

Variable	Percentage Change on Endogenous Variables (Reform 1)								
	educ1	educ2	educ3	educ4	occu1	occu2	occu3	occu4	occu5
$\Delta\xi/\xi(\%)$	-3.3122	-1.835	-2.0567	-2.0934	-1.60402	-1.22948	-3.06843	-2.702	-1.6196
$\Delta\bar{s}_1/\bar{s}_1(\%)$	5.99463	6.2574	7.00643	5.68083	7.1509	6.029	6.692	6.129	4.8644
$\Delta\bar{s}_2/\bar{s}_2(\%)$	5.9939	6.25816	7.01037	5.6822	7.14767	6.0354	6.6932	6.13024	4.8653
$\Delta\bar{s}_3/\bar{s}_3(\%)$	5.9946	6.25746	7.01319	5.6817	7.14637	6.0373	6.6953	6.13015	4.8648
$\Delta\phi/\phi(\%)$	1.3863	1.3466	2.56013	1.69716	1.9111	1.6512	1.75399	1.6458	1.1842
Variable	Percentage Change on Endogenous Variables (Reform 2)								
	educ1	educ2	educ3	educ4	occu1	occu2	occu3	occu4	occu5
$\Delta\xi/\xi(\%)$	-3.2352	-1.5442	-1.91236	-2.0454	-1.12617	-0.9699	-2.9035	-2.6034	-1.797
$\Delta\bar{s}_1/\bar{s}_1(\%)$	6.2912	6.5712	5.8832	5.6224	6.43925	4.3516	7.3383	6.1197	5.0133
$\Delta\bar{s}_2/\bar{s}_2(\%)$	6.2894	6.57154	5.88186	5.6221	6.4425	4.35198	7.34117	6.1209	5.0155
$\Delta\bar{s}_3/\bar{s}_3(\%)$	6.2905	6.5707	5.88235	5.62088	6.43764	4.3504	7.34325	6.1213	5.01285
$\Delta\phi/\phi(\%)$	1.50432	1.74768	2.61037	2.1875	1.7094	2.2089	2.5029	1.9462	1.35935

4.7 CONCLUSION

An empirical version of the model of job search based on Mortensen's (1986) in which the search effort is endogenized has been specified in this paper. We focus on the search behavior of the unemployed workers only. The optimal choices of search intensity and reservation wage are determined simultaneously by maximizing the discounted value of future net income. A higher level of search effort rises the job offer arrival rate but at the meanwhile the cost of search increases as well. If the marginal return to search is too low compared with the marginal cost of search, the unemployed will decide not to search. The optimal search intensity satisfies the marginal cost equals the marginal returns to search condition if the outcome is positive. In addition, the reservation wage and the search intensity move simultaneously. Therefore, a more generous insurance compensation will increase reservation wage and lower the level of search intensity at the same time. We modified Mortensen's (1986) model by the following: first, adding a constant term α_0 to capture the transition into work of non-searchers; second, defining search effort as a vector of three search indicators, $s = (s_1, s_2, s_3)$, as opposed to the one-dimensional search intensity, s, used in the original model; and third, specifying the job offer arrival rate as the composite sum of various search indicators. In comparison, the Mortensen model (1986) has the arrival rate depend on the total search effort s only.

We use the second longitudinal file of Labor Market Activity Survey (LMAS) which provides the information on the labor market participation and job characteristics of all responding individuals over a three-year-long period, 1988-1990. The data contains three indicators of search intensity, one for search or not, one for number of channels used for search and one for amount of time spent on search. In the empirical specification, the observed search indicators are linked to the optimal search intensity derived from the theoretical model. The empirical model is consistent with the structure of the theoretical model: The reservation wage is computed from the reservation wage equation and the values of three search indicators are determined by equating the marginal cost of search to the marginal return to search. The stochastic specification of the empirical model allows for the unobserved heterogeneity in the market-determined part of the job offer arrival rate, reporting errors in the search indicators and measurement error in observed wage offers. The parameters of the model are estimated by applying the simulated maximum likelihood method to deal with the problem of multidimensional integration of normally distributed random variables present in the likelihood function.

The parameter estimates of the structural model reveal that the market-determined part of the job offer arrival rate, λ, decreases with age for the unemployed individuals who exceed the age of 24. The probability of receiving job offers increases with the level of education. Thus, the highly educated workers have more opportunities on the market. In addition, in the more industrialized area (area1 and area3), the unemployed individuals receive more job offers. Finally, the more professional training possessed by the workers, the more job offers they receive. For instance, the engineers (occu2) obtain the highest arrival rate while the clerks (occu3) who are lack of skill-specific training receive the lowest arrival rate. The coefficient estimates of effectiveness of search clearly show that all of the three search indicators influence the job offer arrival rate significantly. The number of channels used for searching has the largest effect on the arrival rate compared with other search indicators. The estimation results for endogenous variables show that the workers who are highly educated, more professional and living in the industrialized area search more and demand higher reservation wages. Consequently, the empirical estimation results suggest that the unemployed individuals who are facing more opportunities on the job market search more intensively due to their higher value of return to search which is consistent with the result predicted by the theoretical model. Moreover, the empirical results also reveal the important impact of search intensity on the labor market transitions: the unemployed with a higher level of search effort and a relatively lower value in reservation wage transit into work sooner compared with the others (Table 4.9).

We simulate the effects, on the behavior of job search and on the transition rate into work for the unemployed, of two alternative policy changes on the insurance compensation system with the same benefit save using the parameters of the structural model. The simulation results show that a lower level of insurance benefits leaves a lower value of reservation wage and a higher level of search intensity at the same time which is consistent with the prediction of the theoretical model. Compared with the permanent cut in the weekly EI benefits, monitoring and benefits sanctions for insufficient search influence the search behavior of those who were less likely to search more effectively. To them, the more obvious improvements in labor market transition rate with benefit sanction are observed. In addition, most of the workers who did search intensively before the reforms are implemented also make more progress in transition rate with benefit sanction. The only exception is the workers in occupation 1 (managers, doctors and etc.) who are better off with permanent benefit cut. As a result, when the number of workers in occupation 1 among the unemployed is not very large, the monitoring and benefit sanction for insufficient search may serve as a more effective way to restore incentives for the unemployed workers to search more and demand less on reservation wage to get back to work soon. The effect of benefit sanctions has not attracted much attention in the literature. Additionally, the empirical evidence is mixed. Our simulation results using Canadian data are consistent with the results by Lollivier and Rioux (2002) using French data which suggests that monitoring and benefit sanctions affect the job search behavior and thus the job finding success of unemployed workers in an important way. However, the simulation results in our study also show that the benefit deduction is less effective to those who are lowly educated and lack of skill-specific training due to their insufficient market opportunities. To them, the government needs to consider to provide more programs of education and training courses for specific skills to improve their situation on the labor market and thus to accelerate their transitions into work.

5. CONCLUSION

In economics, static studies involve analysis focusing only on a particular period of time. On the other hand, dynamic studies focus on the analysis over more than one period. Economic variables usually do not remain unchanged but move over time in reality. Dynamic analysis examines the dynamic paths of endogenous variables when exogenous variables change over time. In addition, dynamic framework allows us to investigate whether or not the endogenous variables converge to a steady state, and how and when the equilibrium is achieved over time.

The study investigates the welfare implications of implementing a dynamic as opposed to static policy choice, by using the application of dynamic economic principles in the applied microeconomic framework, of public, health and labor economics. In the first essay, we examine the optimal choice of commodity tax, when the percentage of consumers purchasing the new substitute good in the population evolves over time, according to the evolutionary game as opposed to the standard Ramsey commodity tax problem in the static framework. In the second essay, we analyze the choice of government funding policy towards communicable diseases, in an economy where there exists monopoly power in the provision of the pharmaceutical drug, and the proportion of sick in the population changes over time. The third essay proposes a non-stationary versus a stationary job search model, to investigate the optimal strategy of unemployed job seekers, when the exogenous variables change over time as opposed to remaining constant.

The study suggests that the incorporation of optimal dynamic agent behavior in microeconomic models has non-trivial welfare implications, suggesting the over usage of static microeconomic models in public, health and labor economics may mask opportunities for welfare improvement through the implementation of dynamic as opposed to static policy choices. The study also extends the traditional Ramsey (1927) static model for optimal commodity taxation to a dynamic framework with an evolutionary game. Additionally, the Mechoulan (2007) model is extended to incorporate the government's role in curtailing the dispersion speed of a communicable disease, when a monopolist supplies the needed pharmaceutical drug. The extension demonstrates that government's role may be appropriate as the prevalence of a communicable disease increases. Furthermore, in a job search environment, insurance compensation has incentive effects for the optimal strategy of job seekers in a dynamic as opposed to static underlining framework. The study reveals that economic dynamics is an important consideration in the public sector choice of appropriate commodity tax policy, the choice of government funding policy for treatment of communicable diseases in the presence of monopoly power, and in determining the optimal job search strategy in labor markets. These results provide an impetus for further work in the application of dynamic economic choices, in public economics, health economics and labor economics.

APPENDIX A

Appendix A.1: Proof of Proposition 2.2.

Suppose at some time t, $U_1(t) = U_2(t)$ where $U_1(t) = V_1(\cdot) + a(t)e_1$ and $U_2(t) = V_2(\cdot) + (1 - a(t))e_2$. If a small percentage of type 2 consumers mistakenly switch to adopt the new technology at time t, the expected utility for consumers choosing good x_1 at time $t+1$ becomes $U_1'(t) = (a(t) + \epsilon)[V_1(\cdot) + e_1] + (1 - a(t) - \epsilon)V_1(\cdot) = V_1(\cdot) + [a(t) + \epsilon]e_1$, where: ϵ is positive but sufficiently small. Whereas, the remaining type 2 consumers who do not deviate at time t will get the expected payoff $U_2'(t) = (a(t) + \epsilon)V_2(\cdot) + (1 - a(t) - \epsilon)[V_2(\cdot) + e_2] = V_2(\cdot) + [1 - a(t) - \epsilon]e_2$ at time $t + 1$. Clearly, $U_1'(t) > U_1(t)$ and $U_2'(t) < U_2(t)$. Since $U_1(t) = U_2(t)$, it follows that $U_1'(t) > U_2'(t)$. Type 2 consumers would like to switch to consume good $x_1(t)$ at time t, so the learning process continues. The equilibrium at $a_3^*(t) = [V_2(\cdot) + e_2 - V_1(\cdot)]/(e_1 + e_2)$ is thus not an ESS. Proposition 2.2 holds.

Appendix A.2: Derivation of Dynamic Commodity Tax Rates for two goods

The government's problem involving the choice of the optimal commodity tax is as follows:

$$(A.1) \quad \max_{\{\tau_1(t),\tau_2(t)\}} \int_0^T e^{-rt}\{a(t)[V_1(q_1(t),p_3,w) + a(t)e_1] + (1-a(t))[V_2(q_2(t),p_3,w) + (1-a(t))e_2]\}\,dt$$

$$s.t. \ (1) \ \sum_1^2 N_i(t)\tau_i(t)x_i(q_i(t),p_3,w) \geq R(t)$$

$$(2) \ a'(t) = g(a(t), q_1(t), q_2(t), p_3, w)$$

where: $q_i(t) = p_i + \tau_i(t)$ for $i = 1, 2$, $a(t) = \frac{N_1(t)}{N}$ and $1 - a(t) = \frac{N_2(t)}{N}$.

Hamiltonian for this optimization problem is defined as: [32]

$$(A.2)\ H(\tau_1(t),\tau_2(t),a(t)) = \{a(t)[V_1(q_1(t),p_3,w)+a(t)e_1]+(1-a(t))[V_2(q_2(t),p_3,w)+(1-a(t))e_2]\} + \mu(t)\{g(a(t),q_1(t),q_2(t),p_3,w)\}$$

where: $\mu(t)$ is the multiplier associated with the replicator dynamics equation which measures the impact of growth in $a(t)$ on the aggregate social welfare (i.e., the shadow price of $a(t)$). The value of $\mu(t)$ is determined by the following differential equation:

$$(A.3)\ \mu'(t) = r\mu(t) - \frac{\partial H(\tau_1(t),\tau_2(t),a(t))}{\partial a(t)} = r\mu(t) - \{(U_1(\cdot)-U_2(\cdot)) + [a(t)e_1-(1-a(t))e_2]\} - \mu(t)\{(1-2a(t))(U_1(\cdot)-U_2(\cdot)) + a(t)(1-a(t))(e_1+e_2)\}$$

where: $U_1(\cdot) = V_1(q_1(t),p_3,w)+a(t)e_1$ and $U_2(\cdot) = V_2(q_2(t),p_3,w)+(1-a(t))e_2$.

The general solution of $\mu(t)$ is derived by integrating Equation A.3 with respect to t: $\mu(t) = \int_t^T e^{-r(s-t)}\frac{\partial H(\tau_1(t),\tau_2(t),a(t))}{\partial a(t)}ds + Ae^{rt}$ where A is an arbitrary constant. Clearly, $\mu(t) > 0$.

The Hamiltonian is maximized subject to the revenue target for the government. From the first order conditions, the following can be derived:

$$(A.4)\ -\left[1-\frac{\alpha_1}{\lambda(t)N}-\tau_1(t)\frac{\partial x_1(q_1(t),p_3,w)}{\partial w}\right]x_1(q_1(t),p_3,w) = \tau_1(t)S_{11} + \left(\frac{\mu(t)}{\lambda(t)N_1(t)}\right)\frac{\partial g(a(t),q_1(t),q_2(t),p_3,w)}{\partial \tau_1(t)}$$

$$(A.5)\ -\left[1-\frac{\alpha_2}{\lambda(t)N}-\tau_2(t)\frac{\partial x_2(q_2(t),p_3,w)}{\partial w}\right]x_2(q_2(t),p_3,w) = \tau_2(t)S_{22} + \left(\frac{\mu(t)}{\lambda(t)N_2(t)}\right)\frac{\partial g(a(t),q_1(t),q_2(t),p_3,w)}{\partial \tau_2(t)}$$

[32] Let c and k represent consumption and wealth respectively. The Hamiltonian $H(c,k)$ can be thought of as a measure of the flow value, in current utility terms, of the consumption-savings combination implied by the consumption choice c, given the predetermined value of k. The Hamiltonian solves the problem of "pricing" saving in terms of current utility by multiplying the flow of saving, $\dot{k} = G(c,k)$, by $J'(k)$, the effect of an increment to wealth on total lifetime utility. A corollary of this observation is that $J'(k)$ has a natural interpretation as the shadow price (or marginal current utility) of wealth. More generally, leaving our particular example aside, $J'(k)$ is the shadow price one should associate with the state variable k (Maurice Obstfeld,1992).

Where $\lambda(t)$ is the Langrangean multiplier associated with the government's budget constraint. It is assumed that prices and income are stationary. Equations (A.4) and (A.5) imply:

$$(A.6) \qquad \tau_1(t)S_{11} = -\theta_1(t)x_1(q_1(t), p_3, w) - \left(\frac{\mu(t)}{\lambda(t)N_1(t)}\right)\frac{\partial g(a(t), q_1(t), q_2(t), p_3, w)}{\partial \tau_1(t)}$$

$$(A.7) \qquad \tau_2(t)S_{22} = -\theta_2(t)x_2(q_2(t), p_3, w) - \left(\frac{\mu(t)}{\lambda(t)N_2(t)}\right)\frac{\partial g(a(t), q_1(t), q_2(t), p_3, w)}{\partial \tau_2(t)}$$

where: $\theta_1(t) = \left[1 - \frac{\alpha_1}{\lambda(t)N} - \tau_1(t)\frac{\partial x_1(q_1(t), p_3, w)}{\partial w}\right]$ and $\theta_2(t) = \left[1 - \frac{\alpha_2}{\lambda(t)N} - \tau_2(t)\frac{\partial x_2(q_2(t), p_3, w)}{\partial w}\right]$ both of which are positive.

The optimal level of $\tau_i(t)$ for $i = 1, 2$ is as follows:

$$(A.8) \qquad \tau_1(t) = \frac{-\theta_1(t)x_1(q_1(t), p_3, w)}{S_{11}} - \left(\frac{1}{S_{11}}\right)\left(\frac{\mu(t)}{\lambda(t)N_1(t)}\right)\frac{\partial g(a(t), q_1(t), q_2(t), p_3, w)}{\partial \tau_1(t)}$$

$$(A.9) \qquad \tau_2(t) = \frac{-\theta_2(t)x_2(q_2(t), p_3, w)}{S_{22}} - \left(\frac{1}{S_{22}}\right)\left(\frac{\mu(t)}{\lambda(t)N_2(t)}\right)\frac{\partial g(a(t), q_1(t), q_2(t), p_3, w)}{\partial \tau_2(t)}$$

APPENDIX B

Appendix B.1: Derivation of Dynamic Prices for Treatment for the Subgame in which the Firm Takes the Offer of the Government and Derivation of Optimal Value of Government Production Subsidy

If the firm decides to accept the government production subsidy at stage 3, its production cost becomes $c_1 - w\theta$ which is represented by $c(w, \theta)$ in the sequel. The demand for treatment at time t is $D(p_t + \tau) = r_t(1 - p_t - \tau)$. Starting at stage 4, given the prevalence path of the disease, the firm maximizes the value of aggregate profit by choosing the optimal dynamic prices for treatment. It is assumed that after time period T, the competitive price applies. The monopolist's problem is summarized as follows:

$$(B.1) \qquad \pi_{c(w,\theta)} \equiv \max_{\{p_t\}} \int_0^T e^{-rt} \{r_t[p_t - c(w,\theta)](1 - p_t - \tau)\}\, dt$$
$$s.t.\ (1)\ r_t' = r_t[\alpha(1 - r_t)(p_t + \tau) - 1]$$

To simplify the notation, let $g(r_t, p_t + \tau)$ represent r_t' in the sequel. Hamiltonian for this optimization problem is defined as: [33]

$$(B.2) \qquad H_2(p_t + \tau, r_t) = r_t[p_t - c(w,\theta)](1 - p_t - \tau) + \mu_2(t) g(r_t, p_t + \tau)$$

[33]Let c and k represent consumption and wealth respectively. The Hamiltonian $H(c, k)$ can be thought of as a measure of the flow value, in current utility terms, of the consumption-savings combination implied by the consumption choice c, given the predetermined value of k. The Hamiltonian solves the problem of "pricing" saving in terms of current utility by multiplying the flow of saving, $\dot{k} = G(c, k)$, by $J'(k)$, the effect of an increment to wealth on total lifetime utility. A corollary of this observation is that $J'(k)$ has a natural interpretation as the shadow price (or marginal current utility) of wealth. More generally, leaving our particular example aside, $J'(k)$ is the shadow price one should associate with the state variable k (Maurice Obstfeld,1992).

where: $\mu_2(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate profit of the firm (i.e., the shadow price of r_t). The value of $\mu_2(t)$ is determined by the following differential equation:

$$(B.3) \qquad \frac{\partial H_2(p_t+\tau, r_t)}{\partial r_t} = [p_t - c(w,\theta)](1-p_t-\tau) + \mu_2(t)\frac{\partial g(r_t, p_t+\tau)}{\partial r_t} = -\mu_2'(t)$$

The general solution of $\mu_2(t)$ is derived by integrating equation B.3 with respect to t: $\mu_2(t) = \int_t^T e^{-r(s-t)}\frac{\partial H_2(p_t+\tau, r_t)}{\partial r_t}ds$. Given the facts that $[p_t - c(w,\theta)](1-p_t-\tau) > 0$ for all $t \in (0,T)$ and the value of $\frac{\partial g(r_t, p_t+\tau)}{\partial r_t}$ decreases with r_t (i.e., $\frac{\partial^2 g(r_t, p_t+\tau)}{\partial r_t^2} = -2\alpha(p_t+\tau) < 0$), the positive impact of the growth in r_t on aggregate profit is declining with r_t. $-\mu_2'(t) > 0$ and approaches zero when r_t is close to 1. $\mu_2(t)$ is obtained by integrating $-\mu_2'(t)$ with respect to t. Therefore, $\mu_2(t) > 0$. The optimal level of dynamic prices for treatment is determined by maximizing the Hamiltonian. The first order condition is derived as follows:

$$(B.4) \qquad \frac{\partial H_2(p_t+\tau, r_t)}{\partial p_t} = r_t[1-2p_t-\tau+c(w,\theta)] + \mu_2(t)\frac{\partial g(r_t, p_t+\tau)}{\partial p_t} = 0$$

The optimal value of p_t for the subgame in which the firm takes the offer of government is thus as follows:

$$(B.5) \qquad p_t = \frac{1}{2}[1 + c(w,\theta) - \tau + \mu_2(t)\alpha(1-r_t)]$$

Given $c(w,\theta) = c_1 - \theta w$ and $\tau = \frac{rw}{1-e^{-rT}}$, the profit function of the firm at time t can thus be obtained by using equation B.5 as:

$$(B.6) \qquad \pi_t = \frac{r_t}{4}\left[\left(1 - c_1 + (\theta - \frac{r}{1-e^{-rT}})w\right)^2 - \alpha^2(1-r_t)^2(\mu_2(t))^2\right]$$

From equation B.6, it can be shown that $\frac{d\pi_t}{d\theta} = \frac{r_t w}{2}\left[1 - c_1 + (\theta - \frac{r}{1-e^{-rT}})w\right] > 0$ if $1 - c_1 + (\theta - \frac{r}{1-e^{-rT}})w > 0$ and $w > 0$. Consequently, the profit of the firm at time t raises with productivity type θ.

In stage 2, the government determines the optimal value of production subsidy provided to the monopolist for cost deduction. Given $p_t(r_t, c_1, w, \theta; a)$, the function of optimal dynamic prices when the firm accepts the offer of subsidy fund, the government's problem involving the choice of the optimal fund is as follows:

$$(B.7) \qquad \max_{\{w\}} E_{c_1} E_\theta \left\{ \int_0^T e^{-rt} \left[r_t (V_1(p_t(r_t, c_1, w, \theta; a) + \tau) + V_1(\tau)) + (1 - r_t) V_2(\tau) \right] dt \right\}$$
$$s.t. \ (1) \ r_t' = r_t \left[\alpha(1 - r_t)(p_t(r_t, c_1, w, \theta; a) + \tau) - 1 \right]$$
$$(2) \ \pi_t(r_t, c_1, w, \theta; a) \geq \pi_t(r_t, c_1, w, \theta; b)$$

where $\tau = \frac{rw}{(1-e^{-rT})}$ is the constant tax paid by every consumer in each time period to raise the money for production subsidy provided by the government. $V_1(p_t + \tau) = \frac{1}{2} - (p_t + \tau) + \frac{1}{2}(p_t + \tau)^2$ is the expected indirect utility for patients who get treated at time t, $V_1(\tau) = -\tau(p_t + \tau)$ is the expected indirect utility for patients who do not purchase treatment at time t and $V_2(\tau) = \frac{1}{2} - \tau$ is the expected utility of consumers who are healthy at time period t. (Remind that the taste parameter for being healthy, β, is uniformly distributed in the interval $[0, 1]$). Clearly, $V_1(p_t + \tau) < V_2(\tau)$ by assuming $p_t + \tau < \min\{1, \sqrt{2p_t}\}$. (2) is the constraint that the firm takes the offer of the government. The Hamiltonian of this optimization problem is defined as:

$$(B.8) H_1(p_t(r_t, c_1, w, \theta; a) + \tau, r_t) = E_{c_1} E_\theta \{ r_t (V_1(p_t(r_t, c_1, w, \theta; a) + \tau) + V_1(\tau)) + (1 - r_t) V_2(\tau)$$
$$+ \mu_1(t) g(r_t, p_t(r_t, c_1, w, \theta; a) + \tau) \}$$

where: $\mu_1(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate social welfare (i.e., the shadow price of r_t). The value of $\mu_1(t)$ is determined by the following differential equation:

$$(B.9)\frac{\partial H_1(p_t(r_t, c_1, w, \theta; a) + \tau, r_t)}{\partial r_t} = E_{c_1}E_\theta\{V_1(p_t+\tau) + V_1(\tau) - V_2(\tau) + r_t(\frac{\partial V_1(p_t+\tau)}{\partial p_t} + \frac{\partial V_1(\tau)}{\partial p_t})\frac{\partial p_t}{\partial r_t}$$
$$+\mu_1(t)(\frac{\partial g(r_t, p_t+\tau)}{\partial r_t} + \frac{\partial g(r_t, p_t+\tau)}{\partial p_t}\frac{\partial p_t}{\partial r_t})\} = -\mu_1'(t)$$

Where: $p_t = p_t(r_t, c_1, w, \theta; a)$. The general solution of $\mu_1(t)$ is derived by integrating equation B.9 with respect to t: $\mu_1(t) = \int_t^T e^{-r(s-t)}\frac{\partial H_1(p_t(r_t, c_1, w, \theta; a) + \tau, r_t)}{\partial r_t}ds$. Since $V_1(p_t+\tau) < V_2(\tau)$ for all applicable p_t and $V_1(\tau) < 0$, $V_1(p_t+\tau) + V_1(\tau) - V_2(\tau) < 0$. Furthermore, the indirect utility of consumers decreases with p_t and $\frac{\partial p_t}{\partial r_t} < 0$, so $\left(\frac{\partial V_1(p_t+\tau)}{\partial p_t} + \frac{\partial V_1(\tau)}{\partial p_t}\right)\frac{\partial p_t}{\partial r_t} > 0$. In addition, $\frac{\partial g(r_t, p_t+\tau)}{\partial r_t}$ declines with r_t and becomes negative for $r_t > \frac{\alpha(p_t+\tau)-1}{2\alpha(p_t+\tau)}$ and $\frac{\partial g(r_t, p_t+\tau)}{\partial p_t}\frac{\partial p_t}{\partial r_t} < 0$. Clearly, $-\mu_1'(t) < 0$ and its absolute value reduces with r_t: the negative impact of growth in r_t on the total social welfare is getting smaller with r_t and approaches zero when r_t is close to 1. As $\mu_1(t)$ is the aggregate sum of $-\mu_1'(t)$ for $t \in (t, T)$, apparently, $\mu_1(t) < 0$.

The monopolist takes the offer of production subsidy if and only if $\theta \geq \frac{r}{(1-e^{-rT})}$, the participation constraint of the firm is thus binding only at $\theta = \frac{r}{(1-e^{-rT})}$. The Hamiltonian is maximized subject to participation constraint of the firm. Given $V_1(p_t+\tau) = \frac{1}{2} - (p_t+\tau) + \frac{1}{2}(p_t+\tau)^2$, $V_1(\tau) = -\tau(p_t+\tau)$, $V_2 = \frac{1}{2} - \tau$, $\pi_t(r_t, c_1, w, \theta; a) = \frac{r_t}{4}[(1 - c_1 + (\theta - \frac{r}{1-e^{-rT}})w)^2 - \alpha^2(1-r_t)^2(\mu_2(t))^2]$ and $\pi_t(r_t, c_1, w, \theta; b) = \frac{r_t}{4}[(1-c_1)^2 - \alpha^2(1-r_t)^2(\mu_3(t))^2]$, the first order condition is thus derived as:

$$(B.10)E_{c_1}E_\theta\{(\frac{\partial p_t}{\partial w} + \frac{\partial \tau}{\partial w})[r_t(p_t+\tau-1) + \mu_1(t)\alpha r_t(1-r_t)] - (1-r_t)\frac{\partial \tau}{\partial w} + r_t[\frac{\partial V_1(\tau)}{\partial p_t}\frac{\partial p_t}{\partial w} + \frac{\partial V_1(\tau)}{\partial \tau}\frac{\partial \tau}{\partial w}]\}$$
$$-\frac{\lambda_1(t)r_t}{2}[1 - c_1 + (\theta - \frac{r}{(1-e^{-rT})})w][\theta - \frac{r}{(1-e^{-rT})}] = 0$$

where $\lambda_1(t)$ is the Lagrangian multiplier associated with the participation constraint of the firm. Clearly, $\lambda_1(t) > 0$ for $\theta = \frac{r}{(1-e^{-rT})}$ and $\lambda_1(t) = 0$ for $\theta > \frac{r}{(1-e^{-rT})}$.

Given $\frac{\partial p_t}{\partial w} = -\frac{1}{2}[\theta + \frac{r}{(1-e^{-rT})}]$, $\frac{\partial \tau}{\partial w} = \frac{r}{(1-e^{-rT})}$, $p_t = \frac{1}{2}[1 + c_1 - (\theta + \frac{r}{(1-e^{-rT})})w + \mu_2(t)\alpha(1-r_t)]$, $\theta \sim U[\underline{\theta}, \overline{\theta}]$ and $c_1 \sim U[\underline{c}, \overline{c}]$, after calculation, the first term of LHS of equation B.10 is equal to $\frac{r_t w}{12}(\overline{\theta}^2 + \overline{\theta}\underline{\theta} + \underline{\theta}^2) + \frac{r_t r(\underline{\theta}+\overline{\theta})w}{4(1-e^{-rT})} - \frac{3r_t r^2 w}{4(1-e^{-rT})^2} - [\frac{r_t(\overline{\theta}+\underline{\theta})}{8} - \frac{r_t r}{4(1-e^{-rT})}](1 + \frac{\underline{c}+\overline{c}}{2} + \alpha(1-r_t)[\mu_2(t) + 2\mu_1(t)]) + \frac{r_t(\underline{\theta}+\overline{\theta})}{4} - \frac{r}{(1-e^{-rT})} - \frac{r_t r}{2(1-e^{-rT})}[\frac{(\underline{c}+\overline{c})}{2} + \mu_2(t)\alpha(1-r_t)]$. Substituting back to equation B.10, the optimal value of the government research grant is thus as follows:

$$(B.11) w = \frac{\left(\frac{\overline{\theta}+\underline{\theta}}{8} - \frac{r}{4(1-e^{-rT})}\right)\left[1 + \frac{\overline{c}+\underline{c}}{2} + \alpha(1-r_t)(\mu_2(t) + 2\mu_1(t))\right] - \frac{\overline{\theta}+\underline{\theta}}{4} + \frac{r}{r_t(1-e^{-rT})} + \frac{r[\frac{\underline{c}+\overline{c}}{2} + \alpha\mu_2(t)(1-r_t)]}{2(1-e^{-rT})} + [\ldots]}{\frac{1}{12}(\overline{\theta}^2 + \overline{\theta}\underline{\theta} + \underline{\theta}^2) + \frac{r(\underline{\theta}+\overline{\theta})}{4(1-e^{-rT})} - \frac{3r^2}{4(1-e^{-rT})^2} - \frac{\lambda_1(t)}{2}(\theta - \frac{r}{(1-e^{-rT})})^2}$$

where: $[\ldots] = \frac{\lambda_1(t)}{2}(1 - c_1)[\theta - \frac{r}{(1-e^{-rT})}]$.

Appendix B.2: Derivation of Dynamic Prices for Treatment for the Subgame in which the Firm Rejects the Offer of the Government

In stage 3, if the monopolist decides not to take the subsidy fund of the government, the production cost of the firm is unchanged, equal to c_1. The firm maximizes the aggregate value of profit by choosing the optimal level of dynamic prices for treatment given the prevalence path of the disease starting at stage 4. The problem of the monopolist is summarized as follows:

$$(B.12) \qquad \pi_{c_1} \equiv \max_{\{p_t\}} \int_0^T e^{-rt}\left\{r_t(p_t - c_1)(1 - p_t)\right\} dt$$
$$s.t.\ (1)\ r_t' = r_t[\alpha(1 - r_t)p_t - 1]$$

Let $g(r_t, p_t)$ represent r_t' in the sequel. Hamiltonian for this optimization problem is defined as:

$$(B.13) \qquad H_3(p_t, r_t) = \{r_t(p_t - c_1)(1 - p_t)\} + \mu_3(t)g(r_t, p_t)$$

where: $\mu_3(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate profit of the firm (i.e., the shadow price of r_t). The value of $\mu_3(t)$ is determined by the following differential equation:

$$(B.14) \qquad \frac{\partial H_3(p_t, r_t)}{\partial r_t} = (p_t - c_1)(1 - p_t) + \mu_3(t)\frac{\partial g(r_t, p_t)}{\partial r_t} = -\mu_3'(t)$$

The general solution of $\mu_3(t)$ is derived by integrating equation B.14 with respect to t: $\mu_3(t) = \int_t^T e^{-r(s-t)} \frac{\partial H_3(p_t, r_t)}{\partial r_t} ds$. Given the facts that $(p_t - c_1)(1 - p_t) > 0$ for all $t \in (0, T)$ and $\frac{\partial g(r_t, p_t)}{\partial r_t}$ decreases with r_t, the positive impact of r_t on aggregate profit is decreasing in r_t and approaches zero eventually (i.e., $-\mu_3'(t) > 0$), clearly, $\mu_3(t) > 0$. The optimal level of dynamic prices for treatment is determined by maximizing the Hamiltonian. The first order condition is derived as follows:

$$(B.15) \qquad \frac{\partial H_3(p_t, r_t)}{\partial p_t} = r_t[1 - 2p_t + c_1] + \mu_3(t)\frac{\partial g(r_t, p_t)}{\partial p_t} = 0$$

The optimal value of p_t for the subgame in which the firm rejects the subsidy fund of government is thus as follows:

$$(B.16) \qquad p_t = \frac{1}{2}[1 + c_1 + \mu_3(t)\alpha(1 - r_t)]$$

Given equations B.3 and B.14, if the ex-post cost of production after receiving the government funding is sufficiently low such that $p_t(r_t, c_1, w, \theta; a) + \tau < p_t(r_t, c_1, w, \theta; b)$, clearly,

$\mu_2(t) > \mu_3(t)$ since $[p_t(r_t, c_1, w, \theta; a) - c(w, \theta)](1 - p_t(r_t, c_1, w, \theta; a) - \tau) > [p_t(r_t, c_1, w, \theta; b) - c_1](1 - p_t(r_t, c_1, w, \theta; b))$.

In addition, the firm's problem for the subgame without government intervention is as same as that for the subgame in which the government involves but the firm rejects the offer provided by the government.

Appendix B.3: Comparison of the Rate of Decrease of the Expected Total Social Welfare with Growth in Proportion of Sick for the Subgames with and without Government Funding

The expected value of total social welfare for the subgame in which the firm takes the offer of production subsidy is represented by $E_{c_1} E_\theta \left\{ \int_0^T e^{-rt} \left[r_t(V_1(p_t(r_t, c_1, w, \theta; a) + \tau) + V_1(\tau)) + (1 - r_t)V_2(\tau) \right] dt \right\}$. While for the subgame in which the government does not involve, expected total social welfare is represented by $E_{c_1} \left\{ \int_0^T e^{-rt} \left[r_t V_1(p_t(r_t, c_1)) + (1 - r_t)V_2 \right] dt \right\}$. As shown in Appendix B.1, the aggregate social welfare declines with the proportion of sick (i.e., $\mu_1(t) < 0$). The rate of decrease of the expected value of total social welfare with the growth in r_t for the subgames with and without subsidy fund of government would be further compared in below. The Hamiltonian for the subgame with government funding is defined as:

$$(B.17) H_1(p_t(r_t, c_1, w, \theta; a) + \tau, r_t) = E_{c_1} E_\theta \{ r_t(V_1(p_t(r_t, c_1, w, \theta; a) + \tau) + V_1(\tau)) + (1 - r_t)V_2(\tau) \\ + \mu_1(t) g(r_t, p_t(r_t, c_1, w, \theta; a) + \tau) \}$$

where: $\mu_1(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate social welfare (i.e., the shadow price of r_t). The value of $\mu_1(t)$ is determined by the following differential equation:

$$(B.18) \frac{\partial H_1(p_t(r_t, c_1, w, \theta; a) + \tau, r_t)}{\partial r_t} = E_{c_1} E_\theta \{ V_1(p_t + \tau) + V_1(\tau) - V_2(\tau) + r_t \left(\frac{\partial V_1(p_t + \tau)}{\partial p_t} + \frac{\partial V_1(\tau)}{\partial p_t} \right) \frac{\partial p_t}{\partial r_t} \\ + \mu_1(t) \left(\frac{\partial g(r_t, p_t + \tau)}{\partial r_t} + \frac{\partial g(r_t, p_t + \tau)}{\partial p_t} \frac{\partial p_t}{\partial r_t} \right) \} = -\mu_1'(t)$$

Given $V_1(p_t+\tau) = \frac{1}{2} - (p_t+\tau) + \frac{1}{2}(p_t+\tau)^2$, $V_1(\tau) = -\tau(p_t+\tau)$, $V_2 = \frac{1}{2} - \tau$, $p_t = \frac{1}{2}[1 + c(w,\theta) - \tau + \mu_2(t)\alpha(1-r_t)]$ and $g(r_t, p_t+\tau) = r_t\left[\alpha(1-r_t)(p_t+\tau) - 1\right]$, equation B.18 can be rewritten as:

$$(B.19)\frac{\partial H_1(p_t(r_t,c_1,w,\theta;a)+\tau,r_t)}{\partial r_t} = E_{c_1}E_\theta\{\frac{1}{2}[p_t(r_t,c_1,w,\theta;a)+\tau]^2 - p_t(r_t,c_1,w,\theta;a) - \frac{1}{2}\tau^2 - \frac{1}{2}\alpha r_t\mu_2(t) \times[p_t(r_t,c_1,w,\theta;a)-1] + \mu_1(t)[\alpha(p_t(r_t,c_1,w,\theta;a)+\tau)(1-2r_t) - \frac{1}{2}\alpha^2\mu_2(t)r_t(1-r_t)]\} = -\mu_1'(t)$$

The general solution of $\mu_1(t)$ is derived by integrating equation B.19 with respect to t: $\mu_1(t) = \int_t^T e^{-r(s-t)}\frac{\partial H_1(p_t(r_t,c_1,w,\theta;a)+\tau,r_t)}{\partial r_t}ds$. In contrast, the Hamiltonian for the subgame without government production subsidy is defined as:

$$(B.20) \qquad H_4(p_t(r_t,c_1),r_t) = E_{c_1}\left\{r_tV_1(p_t(r_t,c_1)) + (1-r_t)V_2 + \mu_4(t)g(r_t,p_t(r_t,c_1))\right\}$$

where: $\mu_4(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate social welfare (i.e., the shadow price of r_t). The value of $\mu_4(t)$ is determined by the following differential equation:

$$(B.21)\frac{\partial H_4(p_t(r_t,c_1),r_t)}{\partial r_t} = E_{c_1}\left\{V_1(p_t) - V_2 + r_t(\frac{\partial V_1(p_t)}{\partial p_t}\frac{\partial p_t}{\partial r_t}) + \mu_4(t)(\frac{\partial g(r_t,p_t)}{\partial r_t} + \frac{\partial g(r_t,p_t)}{\partial p_t}\frac{\partial p_t}{\partial r_t})\right\} = -\mu_4'(t)$$

Given $V_1(p_t) = \frac{1}{2} - p_t + \frac{1}{2}p_t^2$, $V_2 = \frac{1}{2}$, $p_t = \frac{1}{2}[1 + c_1 + \mu_3(t)\alpha(1-r_t)]$ and $g(r_t,p_t) = r_t\left[\alpha(1-r_t)p_t - 1\right]$, equation B.21 can be rewritten as:

$$(B.22)\frac{\partial H_4(p_t(r_t,c_1),r_t)}{\partial r_t} = E_{c_1}\{\frac{1}{2}[p_t(r_t,c_1)]^2 - p_t(r_t,c_1) - \frac{1}{2}\alpha r_t\mu_3(t)[p_t(r_t,c_1)-1] + \mu_4(t)[\alpha p_t(r_t,c_1)(1-2r_t) - \frac{1}{2}\alpha^2\mu_3(t)r_t(1-r_t)]\} = -\mu_4'(t)$$

The general solution of $\mu_4(t)$ is derived by integrating equation B.22 with respect to t: $\mu_4(t) = \int_t^T e^{-r(s-t)}\frac{\partial H_4(p_t(r_t,c_1),r_t)}{\partial r_t}ds.$

The value of $\mu_1(t)$ and $\mu_4(t)$ could be compared through equations B.19 and B.22. Let us assume that the marginal cost of production can be reduced sufficiently after taking the production subsidy of government such that $p_t(r_t, c_1, w, \theta; a) + \tau < p_t(r_t, c_1)$. First, given $p_t(r_t, c_1, w, \theta; a) + \tau < 1$, $p_t(r_t, c_1) < 1$ and $p_t(r_t, c_1, w, \theta; a) + \tau < p_t(r_t, c_1)$, clearly, $\frac{1}{2}[p_t(r_t, c_1, w, \theta; a)+\tau]^2 - [p_t(r_t, c_1, w, \theta; a)+\tau] > \frac{1}{2}[p_t(r_t, c_1)]^2 - p_t(r_t, c_1)$. Then it is easy to verify that $\frac{1}{2}[p_t(r_t, c_1, w, \theta; a)]^2 - p_t(r_t, c_1, w, \theta; a) - \frac{1}{2}\tau^2 > \frac{1}{2}[p_t(r_t, c_1, w, \theta; a)+\tau]^2 - [p_t(r_t, c_1, w, \theta; a)+\tau]$ given $p_t(r_t, c_1, w, \theta; a) + \tau < 1$. As a result, $\frac{1}{2}[p_t(r_t, c_1, w, \theta; a)]^2 - p_t(r_t, c_1, w, \theta; a) - \frac{1}{2}\tau^2 > \frac{1}{2}[p_t(r_t, c_1)]^2 - p_t(r_t, c_1)$. Second, as $\mu_2(t) > \mu_3(t)$ if $p_t(r_t, c_1, w, \theta; a) + \tau < p_t(r_t, c_1)$, apparently, $-\frac{1}{2}\alpha r_t \mu_2(t)[p_t(r_t, c_1, w, \theta; a) - 1] > -\frac{1}{2}\alpha r_t \mu_3(t)[p_t(r_t, c_1) - 1]$. Thus, the negative impact of the increase in r_t on the current social welfare is smaller for the subgame in which the firm accepts the subsidy fund. Last, for $r_t < \frac{1}{2}$ such that $[\alpha p_t(1-2r_t) - \frac{1}{2}\alpha^2 \mu_i(t) r_t(1-r_t)] > 0$, $i = 2, 3$, clearly, $\mu_1(t)[\alpha(p_t(r_t, c_1, w, \theta; a)+\tau)(1-2r_t) - \frac{1}{2}\alpha^2\mu_2(t) r_t(1-r_t)] > \mu_4(t)[\alpha p_t(r_t, c_1)(1-2r_t) - \frac{1}{2}\alpha^2 \mu_3(t) r_t(1-r_t)]$ given $p_t(r_t, c_1, w, \theta; a) + \tau < p_t(r_t, c_1)$, $\mu_2(t) > \mu_3(t)$, $\mu_1(t) < 0$ and $\mu_4(t) < 0$. While for $r_t \geq \frac{1}{2}$ such that $[\alpha p_t(1-2r_t) - \frac{1}{2}\alpha^2 \mu_i(t) r_t(1-r_t)] < 0$, $i = 2, 3$, $\mu_1(t)[\alpha(p_t(r_t, c_1, w, \theta; a) + \tau)(1-2r_t) - \frac{1}{2}\alpha^2 \mu_2(t) r_t(1-r_t)] < \mu_4(t)[\alpha p_t(r_t, c_1)(1-2r_t) - \frac{1}{2}\alpha^2 \mu_3(t) r_t(1-r_t)]$. Compared with the subgame without government intervention, the growth in r_t has less negative impact when $r_t < \frac{1}{2}$ and more negative impact when $r_t \geq \frac{1}{2}$ on the future total social welfare for the subgame with government production subsidy. However, the influence of r_t on the current social welfare dominates the influence of r_t on the future total social welfare, therefore, $-\mu_4'(t) < -\mu_1'(t) < 0$ for any $r_t \in (0, 1)$. Since $\mu_1(t)$ and $\mu_4(t)$ are obtained by integrating $-\mu_1'(t)$ and $-\mu_4'(t)$ with respect to t respectively, apparently, $\mu_4(t) < \mu_1(t) < 0$. The expected value of total social welfare for the subgame with government funding declines with r_t at a lower speed than that does for the subgame without government intervention.

Appendix B.4: Research Effort towards Impact of Government Fund on Social Welfare Improvement through Funding a Potential Entrant to Reduce Its Fixed Entry Cost

2. Model

Suppose there is a drug monopolist in the market, firm 1, producing treatment with marginal cost c_1. A potential entrant, firm 2, producing at the same marginal cost (i.e., $c_1 = c_2$), faces a fixed entry cost, k_1. The government faces a choice of whether or not to provide a fund G to the potential entrant aimed at reducing its fixed cost of entry. With the government funding, firm 2 could enter the market at time $t \geq T_1$ with ex-post cost of entry $k = k_1 - G$ where $0 < T_1 < T$. Otherwise, firm 2 could not enter, firm 1 remaining to be the monopolist in the market up to time T at which the competitive price applies.

The economy is represented by a sequential game in a dynamic environment. The sequence of the game is as follows: In stage 1, the government chooses whether or not to offer a fund to the potential entrant to reduce its fixed cost of entry. If the government decides to offer the fund, the level of government funding is determined at stage 2. Observing the level of government fund $G > 0$, in stage 3, the incumbent monopolist in the market, firm 1, chooses whether or not to deter the entry of firm 2. If firm 1 decides to deter the entry, it sets the price at the output limit price level in the first period for $t \in [0, T_1)$ to block the entry of firm 2 at time $t \geq T_1$. While if the monopolist decides to accommodate the entry of firm 2, it charges the monopolist price in period 1. In stage 4, firm 2 decides whether or not to enter in the second period (i.e,. for time $t \in [T_1, T))$ given the price charged by the monopolist in the first period. It is assumed that Stackelberg game applies in period 2 when firm 1 sets the price at the output limit price level in the first period. Whereas firm 1 and firm 2 proceed with Cournot competition in the second period if firm 2 decides to enter after observing the monopoly price charged by firm 1 in period 1. In contrast, if the government decides not to provide the fund to firm 2 in the first stage, firm 2 could not enter the market. Therefore, starting at stage 2, the incumbent monopolist in the market, firm 1, chooses dynamic prices for treatment for $t \in [0, T)$ by maximizing its aggregate profit.

The sequential game is solved through backward induction. For the subgame in which the government decides to offer the fund, at stage 4, firm 2 chooses whether or not to enter in period 2 given the prices charged by the monopolist in period 1. In stage 3, observing $G > 0$, firm 1 chooses to either deter the entry or accommodate the entry of firm 2 by comparing its total profits between charging the output limit price and the monopolist price in the first period. The level of government funding G is determined by maximizing the expected value of total social welfare by the government in stage 2. While for the subgame without government involvement, firm 2 could not enter the market. Firm 1 chooses the monopoly prices for treatment dynamically starting at stage 2. In what follows, the subgame with government funding is first solved and the subgame without government intervention is next. The government choice of whether or not to offer a fund to the new entrant for reducing its fixed cost of entry takes place in stage 1 of the game. Given the assumptions, clearly, the equilibria of the game are as follows: For the subgame in which the level of government funding $G > 0$, firm 2 enters in the second period if firm 1 charges the monopoly price in the first period, and firm 2 does not enter if firm 1 sets the price at the output limit price level in the first period. If we further assume that the total profit of firm 1 by setting the price at the output limit price level exceeds that by charging the monopoly price in period 1, the only equilibrium when the government funding $G > 0$ becomes that firm 2 does not enter given the monopolist charges the output limit price in period 1. Whereas for the subgame in which the value of government funding $G = 0$, firm 2 could not enter, firm 1 being the monopolist in the market for $t \in [0, T)$.

2.1. Subgame with Government Funding. When the government decides to offer the fund to potential entrant firm 2, the fixed cost of entry of firm 2 is reduced which is represented as $k = k_1 - G$. At stage 4, firm 2 chooses whether or not to enter at $t \geq T_1$ given the prices charged by the monopolist in the first period. It is assumed that the stackelberg game applies if firm 1 chooses the output limit price in period 1. Additionally, the firms proceed with Cournot competition in period 2 if firm 2 decides to enter by observing the monopoly price in period 1. In stage 3, observing the government funding to firm 2 is greater than zero (i.e., $G > 0$), firm 1 chooses between the output limit price and the monopoly price. Let $\pi_{1t}(p_{1t}^L)$ and $\pi_{1t}(p_{1t}^{Mg})$ denote the profits of firm 1 at time t by setting the price at the output limit level and at the monopoly level in period 1 respectively when $G > 0$. Firm 2 would enter if the incumbent charged the monopoly price in the first period and firm 2 would not enter if the monopoly charged the output limit price in period 1. Therefore, firm 1 chooses the output limit price in the first period if $\int_0^{T_1} e^{-rt}[\pi_{1t}(p_{1t}^L)]dt + \int_{T_1}^{T} e^{-rt}[\pi_{1t}(p_{1t}^L)]dt \geq \int_0^{T_1} e^{-rt}[\pi_{1t}(p_{1t}^{Mg})]dt + \int_{T_1}^{T} e^{-rt}[\pi_{1t}(p_{1t}^{2Cg})]dt$ where $\pi_{1t}(p_{1t}^{2Cg})$ denotes the profit of firm 1 as a result of Cournot competition when firm 2 enters in the second period.

Given the government funding $G > 0$, if firm 1 decides to block the entry of firm 2 in stage 3, it sets the price at the output limit level in period 1. As a result, in stage 4, firm 2 decides not to enter in the second period by observing, p_{1t}^L, firm 1 charged in the first period. The Stackelberg game is described through backwards induction as follows: In the second period, observing price of firm 1 in the first period, p_{1t}^L, and therefore given the quantity produced by firm 1 in period 1, $q_1(p_{1t}^L)$, firm 2 chooses q_2 to maximize the profit in period 2 as:

$$(B.23) \qquad \max_{q_2} \pi_2(q_1(p_{1t}^L), q_2) \equiv [a - b(q_1(p_{1t}^L) + q_2) - c_2]q_2 - (k_1 - G)$$

From the first order condition, the reaction function of firm 2 is derived as:

$$(B.24) \qquad R_2(q_1(p_{1t}^L)) = \frac{(a - c_2 - bq_1(p_{1t}^L))}{2b}$$

Substituting $R_2(q_1(p_{1t}^L))$ into the equation of B.23, the optimal profit function of firm 2 in the second period is written as:

$$(B.25) \qquad \pi_2(q_1(p_{1t}^L), q_2) = \frac{(a - c_2 - bq_1(p_{1t}^L))^2}{4b} - (k_1 - G)$$

In the first period, given firm 2's reaction function $R_2(q_1(p_{1t}^L))$, firm 1 sets price at the level of p_{1t}^L such that $\pi_2(q_1(p_{1t}^L), q_2) = 0$, the output limit quantity produced by firm 1 is thus determined as:

$$(B.26) \qquad q_1(p_{1t}^L) = \frac{(a - c_2 - 2\sqrt{b(k_1 - G)})}{b}$$

Since at output limit price p_{1t}^L, firm 2 is indifferent between enter or not enter at time $t \geq T_1$ (i.e., $\pi_2(q_1(p_{1t}^L), q_2) = 0$), $q_2 = 0$ at p_{1t}^L. Firm 1 blocks the entry of firm 2 by setting the price at the output limit price level in period 1. Therefore, the total quantity produced at price p_{1t}^L is $Q(p_{1t}^L) = q_1(p_{1t}^L) = \dfrac{(a - c_2 - 2\sqrt{b(k_1 - G)})}{b}$. A constant tax rate τ is collected per time period to compensate the value of government funding G: $\int_0^T e^{-rT}\tau dt = G$. The demand for treatment at price p_{1t}^L is $D(p_{1t}^L) = r_t(1 - p_{1t}^L - \tau)$. Let $D(p_{1t}^L) = Q(p_{1t}^L)$, the output limit price is thus determined as follows:

$$(B.27) \qquad p_{1t}^L = 1 - \tau - \frac{(a - c_2 - 2\sqrt{b(k_1 - G)})}{br_t}$$

Where $\tau = \dfrac{rG}{(1 - e^{-rT})}$. The impacts of government funding G and proportion of sick r_t on the level of output limit price can be derived from equation B.27 as follows:

$$(B.28) \qquad \frac{dp_{1t}^L}{dG} = -\frac{1}{r_t\sqrt{b(k_1-G)}} - \frac{r}{(1-e^{-rT})}$$

And

$$(B.29) \qquad \frac{dp_{1t}^L}{dr_t} = \frac{(a-c_2-2\sqrt{b(k_1-G)})}{br_t^2}$$

In addition, the influence of fixed entry cost of firm 2, k_1, on the output limit price p_{1t}^L is derived as:

$$(B.30) \qquad \frac{dp_{1t}^L}{dk_1} = \frac{1}{r_t\sqrt{b(k_1-G)}}$$

Clearly, given $0 \le r_t \le 1$, $\frac{dp_{1t}^L}{dG} < 0$ and $\frac{dp_{1t}^L}{dk_1} > 0$. The higher fixed entry cost faced by firm 2, the higher price firm 1 could charge to deter the entry of firm 2. And the higher level of government funding G makes the output limit price p_{1t}^L lower. Additionally, since $q_1(p_{1t}^L) > 0$, $a - c_2 - 2\sqrt{b(k_1-G)} > 0$. Therefore, $\frac{dp_{1t}^L}{dr_t} > 0$ which implies that the output limit price increases with the number of sick in the population.

Given equation B.27, the profit function of firm 1 at time t by charging output limit price p_{1t}^L is derived as:

$$(B.31)\pi_{1t}(p_{1t}^L) = r_t(1-p_{1t}^L-\tau)(p_{1t}^L-c_1) = \left(\frac{(a-c_2-2\sqrt{b(k_1-G)})}{b}\right)\left[1-\tau-c_1-\frac{(a-c_2-2\sqrt{b(k_1-G)})}{br_t}\right]$$

Firm 2 does not enter if the incumbent firm 1 sets the price at the output limit price. Therefore, firm 1's total profit by charging p_{1t}^L in the first period is: $\int_0^{T_1} e^{-rt}[\pi_{1t}(p_{1t}^L)]dt + \int_{T_1}^{T} e^{-rt}[\pi_{1t}(p_{1t}^L)]dt$.

In contrast, observing the value of government funding to firm 2, if firm 1 chooses the monopoly price in stage 3, firm 2 enters in stage 4. Two firms proceed with Cournot competition in period 2 then. The game is solved backwards as follows: In the second period, given q_2, the quantity produced by firm 2, firm 1 chooses q_1 to maximize its profit in period 2 as:

$$(B.32) \qquad \max_{q_1} \pi_1(q_1(p_{1t}^{2Cg}), q_2(p_{1t}^{2Cg})) \equiv [a - b(q_1(p_{1t}^{2Cg}) + q_2(p_{1t}^{2Cg})) - c_1]q_1(p_{1t}^{2Cg})$$

From the first order condition, the reaction function of firm 1 to q_2 is derived as:

$$(B.33) \qquad R_1(q_2(p_{1t}^{2Cg})) = \frac{(a - c_1 - bq_2(p_{1t}^{2Cg}))}{2b}$$

Similarly, given q_1, the quantity produced by firm 1, firm 2 chooses q_2 by maximizing its profit as:

$$(B.34) \qquad \max_{q_2} \pi_2(q_1(p_{1t}^{2Cg}), q_2(p_{1t}^{2Cg})) \equiv [a - b(q_1(p_{1t}^{2Cg}) + q_2(p_{1t}^{2Cg})) - c_2]q_2(p_{1t}^{2Cg}) - (k_1 - G)$$

From the first order condition, the reaction function of firm 2 to q_1 is derived as:

$$(B.35) \qquad R_2(q_1(p_{1t}^{2Cg})) = \frac{(a - c_2 - bq_1(p_{1t}^{2Cg}))}{2b}$$

By assuming $c_1 = c_2$, the quantities produced by both firms are equivalent as:

$$(B.36) \qquad q_i(p_{1t}^{2Cg}) = \frac{(a - c_i)}{3b} \qquad \forall\, i = 1, 2$$

Therefore, the total quantity of Cournot competition is $Q(p_{1t}^{2Cg}) = q_1(p_{1t}^{2Cg}) + q_2(p_{1t}^{2Cg}) = \frac{2(a-c_1)}{3b}$. The demand for treatment at price p_{1t}^{2Cg} is $D(p_{1t}^{2Cg}) = r_t(1-p_{1t}^{2Cg}-\tau)$. By setting $D(p_{1t}^{2Cg}) = Q(p_{1t}^{2Cg})$, the price of Cournot competition is derived as:

$$(B.37) \qquad p_{1t}^{2Cg} = 1 - \tau - \frac{2(a-c_1)}{3br_t}$$

Given equation B.37, the profit function of firm 1 at time t by charging the price p_{1t}^{2Cg} is derived as:

$$(B.38) \qquad \pi_{1t}(p_{1t}^{2Cg}) = r_t(1 - p_{1t}^{2Cg} - \tau)(p_{1t}^{2Cg} - c_1) = \left(\frac{2(a-c_1)}{3b}\right)\left[1 - \tau - c_1 - \frac{2(a-c_1)}{3br_t}\right]$$

In stage 3, firm 1 decides to accommodate the entry of firm 2. Therefore, in the first period, the monopoly price for treatment is determined by maximizing the profit of firm 1 as follows:

$$(B.39) \qquad \max_{q_1} \pi_1(q_1(p_{1t}^{Mg})) \equiv [a - bq_1(p_{1t}^{Mg}) - c_1]q_1(p_{1t}^{Mg})$$

From the first order condition, the optimal quantity produced by firm 1 at monopoly price is derived as:

$$(B.40) \qquad q_1(p_{1t}^{Mg}) = \frac{(a-c_1)}{2b}$$

Therefore, the total quantity produced by the monopolist at price p_{1t}^{Mg} is $Q(p_{1t}^{Mg}) = q_1(p_{1t}^{Mg}) = \frac{(a-c_1)}{2b}$. The demand for treatment at price p_{1t}^{Mg} is $D(p_{1t}^{Mg}) = r_t(1 - p_{1t}^{Mg} - \tau)$. The monopoly price is determined by setting $D(p_{1t}^{Mg}) = Q(p_{1t}^{Mg})$. Thus, the function of monopoly price for firm 1 is written as:

$$(B.41) \qquad p_{1t}^{Mg} = 1 - \tau - \frac{(a - c_1)}{2br_t}$$

Given equation B.41, the profit function of firm 1 by charging the monopoly price p_{1t}^{Mg} at time t is derived as:

$$(B.42) \qquad \pi_{1t}(p_{1t}^{Mg}) = r_t(1 - p_{1t}^{Mg} - \tau)(p_{1t}^{Mg} - c_1) = \left(\frac{(a - c_1)}{2b}\right)\left[1 - \tau - c_1 - \frac{(a - c_1)}{2br_t}\right]$$

In stage 3, firm 1 chooses the output limit price as opposed to the monopoly price in the first period if and only if $\int_0^{T_1} e^{-rt}[\pi_{1t}(p_{1t}^L)]dt + \int_{T_1}^{T} e^{-rt}[\pi_{1t}(p_{1t}^L)]dt \geq \int_0^{T_1} e^{-rt}[\pi_{1t}(p_{1t}^{Mg})]dt + \int_{T_1}^{T} e^{-rt}[\pi_{1t}(p_{1t}^{2Cg})]dt$. Given equations B.31, B.38 and B.42, clearly, this condition holds when $b(k_1 - G)$ is sufficiently small. Therefore, assuming $b(k_1 - G)$ is sufficiently small, the only equilibrium for the subgame in which the government funding G is greater than zero is that firm 1 sets the price at the output limit price level in the first period and firm 2 does not enter in the second period.

Given the equilibrium output limit price, p_{1t}^L, the optimal level of government funding is determined in stage 2. At the market prices for treatment $p_{1t}^L+\tau$, the expected indirect utility of patients who get treated at time t is $V_1(p_{1t}^L + \tau) = \frac{1}{2} + \frac{1}{2}(p_{1t}^L + \tau)^2 - (p_{1t}^L + \tau)$, the expected utility of patients who do not buy treatment at time t is $V_1(\tau) = -\tau(p_{1t}^L+\tau)$ and the expected utility for those who are healthy at time t is $V_2(\tau) = \frac{1}{2} - \tau$. In addition, the prevalence path of the disease is characterized by the equation $r_t' = r_t[\alpha(1-r_t)(p_{1t}^L + \tau) - 1]$. Let $p_{1t}^L(r_t, c_2, k_1, G)$ denote the output limit price in the sequel. The marginal cost of production of firm 2 is uniformly distributed on the interval $[\underline{c}, \overline{c}]$. By assuming that the marginal cost of firm 2 is not known but the fixed cost of entry is known by the government, the government's problem can be summarized as follows:

$$(B.43) \quad \max_{\{G\}} E_{c_2} \left\{ \int_0^T e^{-rt} \left[r_t(V_1(p_{1t}^L(r_t, c_2, k_1, G) + \tau) + V_1(\tau)) + (1 - r_t)V_2(\tau) \right] dt \right\}$$

$$s.t. \ (1) \ r_t' = r_t \left[\alpha(1 - r_t)(p_{1t}^L(r_t, c_2, k_1, G) + \tau) - 1 \right]$$

From the first order condition, the optimal value of government funding is determined by the following equation:

$$(B.44) \left(\frac{a - (\frac{c+\bar{c}}{2}) - 2D}{b} \right) \left[\frac{1}{r_t D} + \frac{r}{(1 - e^{-rT})} \right] + \frac{rG}{(1 - e^{-rT})D} - \frac{r}{(1 - e^{-rT})} - \frac{\mu_1(t)\alpha(1 - r_t)}{D} = 0$$

Where $D = \sqrt{b(k_1 - G)}$. $\mu_1(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the expected value of aggregate social welfare. As shown before, $\mu_1(t) < 0$. The optimal solution for G is very long and complicated so it can not be written down here. Moreover, the derivatives of government funding with respect to proportion of sick r_t, fixed cost of entry k_1 and firm 2's marginal cost of production c_2 also can not be derived explicitly. Therefore, we use simulations to examine the impacts of these variables on the optimal value of government funding G. Given $a = 1$, $b = 1$, $k_1 = 1$, $E(c) \in (0, 1)$, $T = 40$, Figure 3.14 to Figure 3.16 show that how the optimal value of government funding G moves with the proportion of sick, r_t, the fixed entry cost of firm 2, k_1, and the expected marginal cost of production of firm 2, $E(c_2)$ respectively. Clearly, first of all, when proportion of sick r_t is smaller than around 0.6, government fund G declines with r_t. While when the number of sick in the population is greater than 0.6, government funding grows with r_t (Figure 3.14). It can be explained as follows: the government funding aimed at reducing the entry cost of the new entrant could induce the monopolist in the market to lower their price for treatment which could benefit the patients. However, the tax rate collected by the government also makes consumers worse off. The disutility of consumers due to paying for the tax is higher when the number of sick in the population is low. As a result, the level of government fund G decreases with r_t when prevalence of the disease is relatively low (i.e., $r_t < 0.6$). While when the proportion of sick is sufficiently large in the population, more and more consumers benefit from government involvement. Consequently, fund G grows with r_t for $r_t > 0.6$. Second, Figure 3.15 shows that government fund increases with fixed entry cost k_1. Moreover, when r_t is relatively low (i.e., $r_t < 0.6$), with the growth in r_t, G raises with k_1 at a decreasing speed. Whereas when r_t is sufficiently large (i.e., $r_t > 0.6$), G rises with k_1 at a increasing speed with the growth in r_t. This is associated with the fact that the level of government fund G decreases with r_t when $r_t < 0.6$ and increases with r_t when $r_t > 0.6$ as is revealed by Figure 3.14. Finally, Figure 3.16 shows that government fund G raises with the expected value of firm 2's marginal cost of production. Therefore, when the marginal cost of new entrant is expected to be high, the government needs to provide more grant to reduce the fixed cost of entry of firm 2. Similar to that is revealed by Figure 3.15, Figure 3.16 also shows that G raises with $E(c_2)$ at a decreasing speed with the growth in r_t when $r_t < 0.6$ while G rises with $E(c_2)$ at a increasing speed with r_t when $r_t > 0.6$.

2.2. Subgame without Government Funding. When the government decides not to provide the fund to the potential entrant in the first stage, the fixed cost of entry for firm 2 is unchanged, which is equal to k_1. It is assumed that firm 2 could not enter at fixed cost k_1. Therefore, the incumbent monopolist, firm 1, chooses the dynamic prices for treatment starting from stage 2 by maximizing the present value of aggregate profit as follows:

$$(B.45) \qquad \max_{\{p_t\}} \int_0^T e^{-rt} \{r_t(p_t - c_1)(1 - p_t)\}\, dt$$
$$s.t. \ (1) \ r_t' = r_t[\alpha(1 - r_t)p_t - 1]$$

As shown in Appendix B.2, the optimal dynamic prices for treatment by the monopolist is characterized as follows:

$$(B.46) \qquad p_{1t}^M = \frac{1}{2}[1 + c_1 + \mu_3(t)\alpha(1 - r_t)]$$

where: $\mu_3(t)$ is the multiplier associated with the prevalence dynamics equation which measures the impact of growth in r_t on the aggregate profit. As shown in Appendix B.2, $\mu_3(t) > 0$.

Let $p_{1t}^M(r_t, c_1)$ represent the monopoly prices for treatment at equilibrium when government funding $G = 0$. Given $p_{1t}^M(r_t, c_1)$, the expected aggregate social welfare in equilibrium for the subgame without government funding is represented as follows: $E_{c_1}\left\{\int_0^T e^{-rt}[r_t V_1(p_{1t}^M(r_t, c_1)) + (1 - r_t)V_2]dt\right\}$ where $V_1(p_{1t}^M(r_t, c_1)) = \frac{1}{2} + \frac{1}{2}(p_{1t}^M(r_t, c_1))^2 - p_{1t}^M(r_t, c_1)$ denotes the expected indirect utility of patients who buy treatment at time t and $V_2 = \frac{1}{2}$ denotes the expected utility of consumers who are healthy at time t.

At stage 1, the government decides whether or not to offer the fund to the new entrant aimed at reducing its fixed cost of entry. Clearly, the decision depends on the result of the comparison of the expected values of the aggregate social welfare with and without government intervention. The government chooses to provide a fund to firm 2 if and only if the following holds:

$$(B.47) \qquad E_{c_2}\left\{\int_0^T e^{-rt}\left[r_t(V_1(p_{1t}^L(r_t, c_2, k_1, G) + \tau) + V_1(\tau)) + (1 - r_t)V_2(\tau)\right]dt\right\} \geq E_{c_1}\left\{\int_0^T e^{-rt}[r_t V_1(p_{1t}^M(r_t, c_1)) + (1 - r_t)V_2]dt\right\}$$

Given equation B.27, the equilibrium market price for treatment at time $t \in [0, T)$ when the value of government funding is greater than zero, $p_{1t}^L + \tau$, is calculated as: $p_{1t}^L + \tau =$

$1-\dfrac{(a-c_2-2\sqrt{b(k_1-G)})}{br_t}$. While when the value of government funding G is equal to zero, the market prices for treatment in equilibrium for $t \in [0,T)$ are p_{1t}^M. Clearly, when $p_{1t}^L+\tau > p_{1t}^M$, equation B.47 does not hold. As a result, government would not involve if the government funding could not reduce the market price for treatment sufficiently. In contrast, let us consider the case that $p_{1t}^L+\tau < p_{1t}^M$. It is easy to show that $p_{1t}^L+\tau$ increases with k_1, c_2 and r_t and decreases with G [34]. In addition, given equation B.46, apparently, p_{1t}^M increases with c_1 and decreases with r_t [35]. First, given proportion of sick, r_t, when the fixed cost of entry for firm 2 is raised, the market price for treatment with government funding is also increased since $\frac{d(p_{1t}^L+\tau)}{dk_1} > 0$. However, at the meanwhile, a higher level of k_1 induces a higher level of fund from the government (as revealed by Figure 3.15) which could make the price for treatment lower (i.e.,$\frac{d(p_{1t}^L+\tau)}{dG} < 0$). Given $\frac{d(p_{1t}^L+\tau)}{dk_1} = \frac{1}{r_t\sqrt{b(k_1-G)}}$ and $\frac{d(p_{1t}^L+\tau)}{dG} = -\frac{1}{r_t\sqrt{b(k_1-G)}}$, clearly, the value of market price for treatment with government funding would not be changed with the growth in fixed entry cost k_1 if the raise in government funding G correspondingly is just equal to the raise in k_1. Therefore, the influence of government funding aimed at reducing the entry cost of the potential entrant on the improvement in social welfare may not be affected by the changes in the level of fixed cost of entry. Second, when the expected marginal cost of production for firms is higher ($E(c_1) = E(c_2) = E(c)$), the market prices for treatment are also expected to be higher (i.e., $\frac{d(p_{1t}^L+\tau)}{dc_2} > 0$, $\frac{dp_{1t}^M}{dc_1} > 0$). However, as revealed by Figure 3.16, the level of fund G rises with the growth in the value of $E(c)$. Therefore, for the subgame with government funding, market prices for treatment $p_{1t}^L+\tau$ could be reduced by the higher level of government funding which would benefit the consumers. Finally, market prices for treatment with government fund, $p_{1t}^L+\tau$, rise with

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[34] $\frac{d(p_{1t}^L+\tau)}{dk_1} = \frac{1}{r_t\sqrt{b(k_1-G)}} > 0$, $\frac{d(p_{1t}^L+\tau)}{dc_2} = \frac{1}{br_t} > 0$, $\frac{d(p_{1t}^L+\tau)}{dr_t} = \frac{a-c_2-2\sqrt{b(k_1-G)}}{br_t^2} > 0$ and $\frac{d(p_{1t}^L+\tau)}{dG} = -\frac{1}{r_t\sqrt{b(k_1-G)}} < 0$.

[35] $\frac{dp_{1t}^M}{dc_1} = \frac{1}{2} > 0$ and $\frac{dp_{1t}^M}{dr_t} = -\frac{1}{2}\mu_3(t)\alpha < 0$.

the prevalence of the disease, r_t. While the monopoly prices without fund, p_{1t}^M, decline with proportion of sick r_t (i.e., $\frac{d(p_{1t}^L+\tau)}{dr_t} > 0$ and $\frac{dp_{1t}^M}{dr_t} < 0$). Figure 3.14 shows that government fund G declines with r_t for $r_t < 0.6$ and G increases with r_t for $r_t > 0.6$. Therefore, at a sufficiently large r_t, prices for treatment $p_{1t}^L + \tau$ could be lowered by a high level of fund G. In addition, the utility loss of consumers associated with paying for the taxes becomes lower when the level of r_t is high. Consequently, government may consider to involve by offering a fund to the new entrant when the marginal costs of production for the firms are expected to be high and the proportion of sick in the population is sufficiently large. Empirical applications for the theoretical model are also conducted. It shows the consistency with the predictions implied by the theoretical framework. However, for the scope of this paper, the empirical examples are not provided here.

APPENDIX C

Appendix C.1: Sample Questions Related to the Job Search in the Survey Data

The construction of the spell of unemployment and of the employment duration is based on the questions like:

"When did you last work?" And

"When did you start working for the current employer?"

And the information on search behavior of the survey respondents is obtained from various questions below:

"In the last 4 weeks ending last Sunday, did you do anything to find work?"

If the answer is "yes", some additional questions are asked:

"What did you do to find work in those 4 weeks?"

"Did you do anything else to find work?"

"As of last week, how many weeks had you been looking for work since the day last worked?"

And

"What was your main activity before you started looking for work?"

Appendix C.2: Construction of Search Channel Index

The search channel index is constructed as below:

Channel	Points
Checked with/Registered at: C.E.C.	1
Checked with other public employment agency	1
Checked with private employment agency	1
Checked with union	1
Checked with employers directly	1
Checked with friends or relatives	1
Placed or answered job ads	1
Looked at job ads	1
Other	1

There are four questions concerning the channels used for looking for work. For each question, the respondents are allowed to choose one out of nine methods for job searching. Thus, the maximum number of search channels reported is four and for each channel they marked they got one point: $s_2^o \in [0, 4]$.

And the search time index is constructed as:

Time (t)	Points
$t = 0$	0
$0 < t \leq 0.25$	1
$0.25 < t \leq 0.5$	2
$0.5 < t \leq 0.75$	3
$0.75 < t \leq 1$	4

Where t is defined as the fraction of time spent on searching in the whole joblessness duration since the last day worked: $t = \frac{spell\ of\ unemployment}{joblessness\ duration}$. Clearly, $s_3^o \in [0, 4]$.

Appendix C.3: Research Effort towards the Non-stationary Job Search Model

In a stationary structural job search model, exogenous variables like the level of unemployment insurance, the market-determined part of the job offer arrival rate and the wage offer distribution are assumed to be unchanging over the spell of unemployment. But in reality, these three variables are very likely to vary with the unemployment duration. Firstly, in most countries, the level of benefits falls dramatically when the unemployed workers exhaust their entitlement to insurance and become assistance recipients. In some countries (as in France from July 1992 to July 2001; European panel survey), the time sequence of insurance benefits itself is declining. Secondly, the market-determined part of arrival rate of job offers which is determined by individual characteristics may also decrease with unemployment duration, due to the "stigma effect" of long-term unemployment. Finally, the distribution of wage offers may be duration-dependent as well. Non-stationarity of the job search behavior arises if one or more of these exogenous variables change over the spell of unemployment, which is very likely. Sooner or later, duration-dependence of exogenous variables is realized and used to determine the optimal strategy of job seekers. Thus, the optimal strategy in a non-stationary job search model is generally not unchanging over time. Consequently, there is a need to model individual's job search process over time based on a non-stationary structural framework. Van den berg (1990a) proposes such a non-stationary theoretical framework to model the movement of reservation wages over the spell of unemployment. In the theoretical part of the paper, non-stationarity originates from the changes over time of three exogenous variables we mentioned above: the level of benefits, the market-determined part of job offer arrival rate and the wage offer distribution. In this very general setting, Van den Berg first shows that the optimal strategy of an unemployed job seeker is still a reservation strategy. However, in his non-stationary theoretical model, he did not make search intensity as a choice variable, and therefore he did not consider the movement over time of search effort, the other optimal choice in the search strategy of an unemployed worker when he/she can influence arrival rate of job offers by varying search intensity. The present study addresses this research gap. [36]

A non-stationary structural model of job search with endogenous search intensity is developed to investigate the simultaneous movements of search intensity and reservation wage over the spell of unemployment for an unemployed worker, or more specifically, when his/her entitlement to EI benefits is decreasing. Due to the scope of this paper, the details of the non-stationary model are deferred to be shown in another paper. But the equations characterizing the optimal search intensity and optimal reservation wage in a non-stationary framework are shown in the sequel.

As in the stationary model, first, the search effort s is defined as a vector of three search indicators: $s = (s_1, s_2, s_3)$ where s_j represents the search indicator by channel j, $j = 1, 2, 3$. Second, we express the job offer arrival rate for individual i who is unemployed, $m_i(s)$, as

[36]Compared with the theoretical framework proposed by Van den berg(1990a), we incorporate the optimal choice of search intensity into the model to investigate the optimal strategy on both reservation wage and search effort of unemployed workers in a non-stationary job search model.

follows: $m_i(s) = [\alpha_1 s_1 + s_1(\alpha_2 s_2 + \alpha_3 s_3)]\lambda_i$, with $\alpha_j > 0, j = 1, 2, 3$ and $\lambda_i > 0$. Apparently, $m_i(s)$ can be rewritten in a vector form as $m_i(s) = (\alpha S')\lambda_i$. Search effort is indicated by S, $S \geq 0$, and S is allowed to be a vector of different combination of search indicator s_1 and other search indicators: $S = (s_1, s_1 s_2, s_1 s_3)$. Search effectiveness parameter $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ measures the impact of search intensity on the arrival rate which is a vector of equal dimension as S. Third, the cost of search function is denoted as $c(s)$ which is a increasing and convex function of its argument. In other words, $c(s)$ has the properties $c'(s) > 0$ *and* $c''(s) > 0$. In addition, corresponding to three indicators of search, the cost of search function is additively separable in search channels: $c(s) = c_0 + \sum_{j=1}^{3} c_j(s_j)$, where c_0 represents the fixed cost of search and $c_j(s_j)$ denotes the cost of search function by channel j, $j = 1, 2, 3$.

Non-stationarity is assumed to be originated from duration dependence of unemployment benefits and of arrival rate of job offers. In Canada, an unemployed worker who is eligible to EI benefits can only receive the benefits up to a maximum limit of weeks which is determined by his/her previous working history and the local rate of unemployment. And when this limit is reached, the worker exhausts his/her entitlement to insurance and becomes an assistance recipient. Let T_i indicate the maximum number of weeks for which the EI benefits may be paid to individual i and t_i denote the elapsed unemployment duration (weeks) of individual i at the survey date. The EI entitlement, τ_i, which is defined as the weeks remaining to EI benefits for individual i when t_i weeks of unemployment has elapsed can be constructed by:

$$(C.1) \qquad \tau_i = T_i - t_i$$

Since we introduce the non-stationarity into the model as a result of the duration-dependence of exogenous variables, it is easy to see that time dependence or duration dependence of exogenous variables can be characterized by their dependence on the EI entitlement τ_i by

Equation $C.1$. To examine the movement of search strategy for an unemployment worker when his/her entitlement to EI benefits is getting less and less, first, we assume that the benefit level per period received by an unemployed worker is a continuous function of τ, the EI entitlement, which is denoted by $b(\tau)$. Second, the market-determined part of job offer arrival rate for unemployed individual i is denoted by $\lambda_i(\tau)$ which is also assumed to be a continuous function of EI entitlement τ. Third, for simplicity, we assume that the wage offer distribution given the reservation wage, $F(w|\xi(\tau))$, is constant over the spell of unemployment. In other words, the variation in wage over the unemployment duration is captured by the movement of reservation wage with EI entitlement τ only. Moreover, the analysis is restricted to the case of no recall of offers received in previous periods. Fourth, in Canada, the unemployed workers who are no more eligible for EI benefits obtain the social assistance benefits only which depends on their household composition and the financial characteristics of other household members. Therefore, after the period that the unemployed exhaust their EI entitlement (when $\tau \leq 0$), the environment remains stationary. Therefore, there is supposed to exist some time T on the duration of unemployment such that all exogenous variables are constant on $[T, \infty)$. Note that, when the spell of unemployment t equals T, the EI entitlement $\tau = 0$. Therefore, this assumption implies that the model reduces to be stationary for $\tau \leq 0$.

As shown in Van den Berg's paper (1991a), the optimal strategy of an unemployed worker is still a reservation strategy when exogenous variables are allowed to change over time. Therefore, it is assumed that the solution of the maximization problem of an individual who is unemployed is again characterized by an optimal choice of the reservation wage $\xi(\tau)$ and the intensity of search $s^*(\tau)$ in a non-stationary framework.

Given above assumptions, an individual who is unemployed maximizes the expected discounted value of future net income over time as described below. In a small time interval

$[t, t+h]$ (or equivalently, in $[\tau - h, \tau]$), an employed worker receives an amount of benefit $b(\tau)h$ less the cost of searching one more period of length h, $c(s^*(\tau))h$. Let $\beta(h)$ denote the discount factor applied to future costs and benefits incurred per period of length h. Under the hypothesis that at most one job offer arrives per period of length h, the market-determined part of the arrival rate of job offers in a small time interval $[t, t+h]$ is $\lambda(\tau)h$. Let $W(w)$ represent the given present value of stopping, accepting the best offer received, w, during any period and then keeping that job forever at wage w. Therefore, the value of searching one more period of length h given that there are τ periods left to the EI entitlement, $V(\tau)$, is given by the following equation:

$$(C.2)\ V(\tau) = \max_{\{s(\tau)\}} \left\{ [b(\tau) - c(s(\tau))]h + \beta(h) \left\{ \begin{array}{l} [\alpha S'(\tau-h)]\lambda(\tau)h \int_{\xi(\tau-h)}^{\infty} [W(x) - V(\tau-h)]dF(x|\xi(\tau-h)) \\ + \{1 - [\alpha S'(\tau-h)]\lambda(\tau)h\}\ V(\tau-h) \end{array} \right\} \right\}$$

in which $c(s(\tau)) = c_0 + \sum_{j=1}^{3} c_j(s_j(\tau))$, $s(\tau) = (s_1(\tau), s_2(\tau), s_3(\tau))$, $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and $S(\tau - h) = (s_1(\tau-h), s_1(\tau-h)s_2(\tau-h), s_1(\tau-h)s_3(\tau-h))$.

After some rearrangements, it can be rewritten as:

$$(C.3)\ V(\tau) - V(\tau-h) + [1-\beta(h)]V(\tau-h) =$$
$$\max_{\{s(\tau)\}} \left\{ [b(\tau) - c(s(\tau))]h + \beta(h)[\alpha S'(\tau-h)]\lambda(\tau)h \int_{\xi(\tau-h)}^{\infty} [W(x) - V(\tau-h)]dF(x|\xi(\tau-h)) \right\}$$

By dividing both sides by h and taking limits as $h \to 0$, one obtains the following continuous time analogue :

$$(C.4) \qquad \frac{dV(\tau)}{d\tau} = b(\tau) - c(s^*(\tau)) + [\alpha S'^*(\tau)]\lambda(\tau) \int_{\xi(\tau)}^{\infty} [W(x) - V(\tau)]dF(x|\xi(\tau)) - \rho V(\tau)$$

in which we assume that $\beta(h) = e^{-\rho h}$, so that the discount factor ρ is defined as $\lim_{h \to 0}[1 - \beta(h)]/h = \rho$.

Assuming the present value of a future earning stream given a wage equal to x is $W(x) = x/\rho$ and together with reservation property $V(\tau) = W(\xi(\tau))$, Equation $C.4$ yields the differential equation which describes the time sequence of optimal reservation wage over EI entitlement τ:

$$(C.5)\ \frac{d\xi(\tau)}{d\tau} = \rho b(\tau) - \rho[c_0 + \sum_{j=1}^{3} c_j(s_j^*(\tau))] + \left\{ \begin{array}{l} [\alpha_1 s_1^*(\tau) + \alpha_2 s_2^*(\tau)s_1^*(\tau) + \alpha_3 s_3^*(\tau)s_1^*(\tau)]\times \\ \lambda(\tau)\int_{\xi(\tau)}^{\infty}[x-\xi(\tau)]dF(x|\xi(\tau)) \end{array} \right\} - \rho\xi(\tau)$$

where we substitute $c(s^*(\tau)) = [c_0 + \sum_{j=1}^{3} c_j(s_j^*(\tau))]$, $S^*(\tau) = [s_1^*(\tau), s_2^*(\tau)s_1^*(\tau), s_3^*(\tau)s_1^*(\tau)]$ and $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ into the Equation $C.4$ and $s_j^*(\tau)$ denotes the optimal search intensity by channel j for unemployed job seekers, $j = 1, 2, 3$. Therefore, the maximization problem of a worker who is unemployed satisfies the differential equation $C.5$. From Equation $C.5$, we can get the expression for $\xi(\tau)$ as below:

$$(C.6) \qquad \rho\xi(\tau) = -\xi'(\tau) + \rho b(\tau) - \rho c(s^*(\tau)) + [\alpha S'^*(\tau)]\lambda(\tau)\int_{\xi(\tau)}^{\infty}[x-\xi(\tau)]dF(x|\xi(\tau))$$

The first order condition for the search intensity choice problem on the RHS. of Equation $C.5$ yields:

$$(C.7)\ \rho c_1'(\bar{s}_1(\tau)) = [\alpha_1 + \alpha_2\bar{s}_2(\tau) + \alpha_3\bar{s}_3(\tau)]\lambda(\tau)\int_{\xi(\tau)}^{\infty}[x-\xi(\tau)]dF(x|\xi(\tau))$$

where $\bar{s}_1(\tau)$ represents the latent search intensity for which the marginal cost of search by means of channel 1 is equal to the marginal returns to search. Therefore:

$$(C.8)\ \rho R_1(\tau) = \rho c_1'(\bar{s}_1(\tau)) = [\alpha_1 + \alpha_2\bar{s}_2(\tau) + \alpha_3\bar{s}_3(\tau)]\lambda(\tau)\int_{\xi(\tau)}^{\infty}[x-\xi(\tau)]dF(x|\xi(\tau))$$

In which $R_1(\tau)$ represents the marginal returns to search by channel 1. And

$$(C.9)\ \rho c_j'(\bar{s}_j(\tau)) = \alpha_j\bar{s}_1(\tau)\lambda(\tau)\int_{\xi(\tau)}^{\infty}[x-\xi(\tau)]dF(x|\xi(\tau))\ \forall\ j = 2, 3\ \mathit{and}$$

$$(C.10)\ \rho R_j(\tau) = \rho c_j'(\bar{s}_j(\tau)) = \alpha_j\bar{s}_1(\tau)\lambda(\tau)\int_{\xi(\tau)}^{\infty}[x-\xi(\tau)]dF(x|\xi(\tau))\ \forall\ j = 2, 3$$

where $\bar{s}_j(\tau)$ denotes the latent search intensity that satisfies the condition of marginal cost of search by channel j equals the marginal returns to search, and $R_j(\tau)$ represents the marginal returns to search by channel j, $j = 2, 3$.

Then the optimal level of search intensity $s_j^*(\tau)$ equals $\max[0, \bar{s}_j(\tau)]$. Thus, the optimal search intensity satisfies the marginal cost of search equals the marginal returns to search condition if $s_j^*(\tau) > 0$.

Equations $C.5$, $C.7$ and $C.9$ can be used to simultaneously determine the reservation wage ξ and the (optimal) latent search intensity $\bar{s}_j$, $j = 1, 2, 3$ as functions of EI entitlement τ. First solve for ξ and $\bar{s}_j$, $j = 1, 2, 3$ at the point T after which all exogenous variables like the EI benefits b and the market-determined job offer arrival rate for individual i, λ_i, are constant. Obviously, the solutions for $\xi(0)$ and $\bar{s}_j(0)$, $j = 1, 2, 3$ (when $t = T, \tau = 0$) are equivalent to the optimal choice of the reservation wage and the search intensity in a stationary model since the environment remains stationary after T. Before T, the $\xi(\tau)$ and $\bar{s}_j(\tau)$, $j = 1, 2, 3$ are continuous functions of τ. Therefore, $\xi(0)$ and $\bar{s}_j(0)$, $j = 1, 2, 3$ serve as an initial condition for the differential equations $C.5$, $C.7$ and $C.9$ in the time interval ending at T within which the exogenous variables $b(\tau)$ and $\lambda_i(\tau)$ are continuous functions of τ. Thus, $\xi(\tau)$ and $\bar{s}_j(\tau)$, $j = 1, 2, 3$ can be solved for every τ in this interval. Using backward induction, the whole sequence of optimal reservation wage $\xi(\tau)$ and the whole sequence of (optimal) latent search effort $\bar{s}_j(\tau)$, $j = 1, 2, 3$ for every $\tau \geq 0$ are then obtained simultaneously by Equations $C.5$, $C.7$ and $C.9$. Clearly, the optimal search strategy for the unemployed job seekers in an non-stationary framework is shown to be consistent with that derived from the stationary job search model.

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